U1H9

1. $f(x) = \dfrac{1}{x-10}$

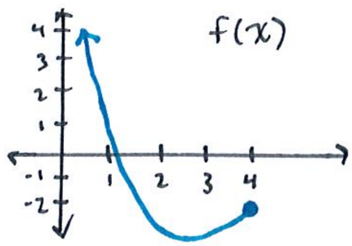
   a. Is f(x) continuous on the interval [0, 15]? Why/why not?

   b. Is f(x) continuous on the interval [-1, 10)?

2. Where is the function discontinuous? What type of discontinuities are they?

$$f(x) = \frac{x^2 - 7x + 10}{x^2 + 2x - 8}$$

3. Given the graph, is f (x) continuous at x = 1 ? Prove.



4. Is g(x) continuous at x = 1 ? Prove.

$$g(x) = \begin{cases} 3x^2, & x < 1 \\ 3, & x = 1 \\ 2x+2, & x > 1 \end{cases}$$

5. Suppose that f and g are continuous functions such that $f(2) = 1$ and $\lim\limits_{x\to 2}[f(x) + 4g(x)] = 13$. Find

   a. $g(2)$

   b. $\lim\limits_{x\to 2} g(x)$

6. Suppose that f and g are continuous functions such that $\lim\limits_{x\to 3} g(x) = 5$ and $f(3) = -2$. Find $\lim\limits_{x\to 3} \dfrac{f(x)}{g(x)}$.

Skip #7

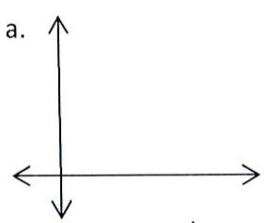
7. In each part sketch a graph of a function f that satisfies the stated conditions.

   a. f is continuous everywhere except at x=3, at which point it is continuous from the right.

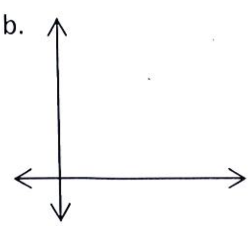
   b. f has a two-sided limit at x = 3, but it is not continuous at that point.

   c. f is continuous on the interval [0,3) and is defined on the closed interval [0,3]; but f is not continuous on the interval [0,3].

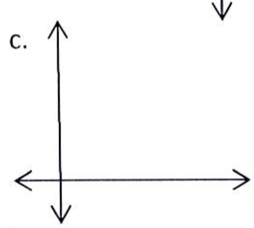
a.



b.



c.



8. Find a value for the constant k, if possible, that will make the function continuous.

   a. $f(x) = \begin{cases} 7x-2, & x \le 1 \\ kx^2, & x > 1 \end{cases}$

   b. $f(x) = \begin{cases} kx^2, & x \le 2 \\ 2x+k, & x > 2 \end{cases}$

Answers

1. a. no, there is a VA at x = 10
   b. yes

2. removable (hole) at x = 2, VA at x = - 4

3. yes. $f(1) = 1 = \lim_{x \to 1} f(x)$

4. no, $g(1) = 3 \neq \lim_{x \to 1} g(x) = \text{DNE}$

Make sure you also prove why the limit as x approaches 1 is DNE!

5. a. 3
   b. 3

6. $-{}^{2}/_{5}$

7. answers vary

8a. k = 5

b. $k = {}^{4}/_{3}$

1. $f(x)=\dfrac{1}{x-10}$

a. Is f(x) continuous on the interval [0, 15]? Why/why not? * no - its not continuous at $x=10$

b. Is f(x) continuous on the interval [-1, 10)?

yes - this interval does not include $x=10$

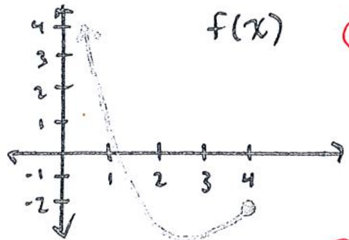
2. Where is the function discontinuous? What type of discontinuities are they?

$$f(x)=\frac{x^2-7x+10}{x^2+2x-8} = \frac{(x-5)\cancel{(x-2)}}{(x+4)\cancel{(x-2)}}$$

$x=2 \rightarrow$ removable

$x=-4 \rightarrow$ infinite

3. Given the graph, is f (x) continuous at x = 4? Prove.



① $\lim_{x\to4} f(x) = -2$

(remember - endpts only have to have a limit exist from 1 side!)

② $f(4) = -2$

③ yes - f(x) is continuous because $\lim_{x\to4} f(x) = f(4)$

4. Is g(x) continuous at x = 1? Prove.

$$g(x)=\begin{cases}3x^2, & x<1\\ 3, & x=1\\ 2x+2, & x>1\end{cases}$$

① $\lim_{x\to1} g(x) = \text{DNE}$

② $f(1) = 3$

③ no - g(x) is not continuous because $\lim_{x\to1} g(x) \neq g(1)$

5. Suppose that f and g are continuous functions such that $f(2)=1$ and $\lim_{x\to2}[f(x)+4g(x)]=13$. Find

a. $g(2)$

$f(2)+4g(2)=13$

$1+4g(2)=13$

$4g(2)=12$

$g(2)=3$

b. $\lim_{x\to2} g(x)$ $= 3$

since we know g(x) is a continuous function

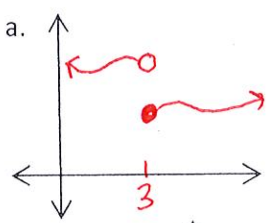
6. Suppose that f and g are continuous functions such that $\lim_{x\to3} g(x)=5$ and $f(3)=-2$. Find $\lim_{x\to3}\dfrac{f(x)}{g(x)}$.

$$\frac{f(3)}{g(3)} = \frac{-2}{5}$$

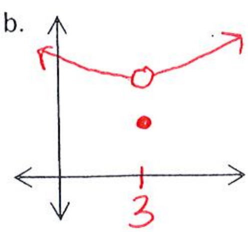
7. In each part sketch a graph of a function f that satisfies the stated conditions. ANSWERS VARY

a. f is continuous everywhere except at x=3, at which point it is continuous from the right.

a.

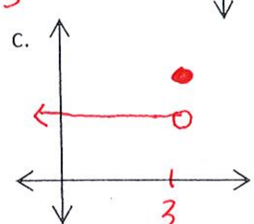


b.



b. f has a two-sided limit at x = 3, but it is not continuous at that point.

c.



c. f is continuous on the interval [0,3) and is defined on the closed interval [0,3]; but f is not continuous on the interval [0,3].

8. Find a value for the constant k, if possible, that will make the function continuous.

a. $f(x)=\begin{cases}7x-2, & x\le1\\ kx^2, & x>1\end{cases}$

$f(1)=7(1)-2$

$=7-2$

$=5$

$kx^2=5$

$k(1)^2=5$

$k=5$

b. $f(x)=\begin{cases}kx^2, & x\le2\\ 2x+k, & x>2\end{cases}$

$f(2)=k(2)^2$

$=4k$

$2x+k=4k$

$2(2)+k=4k$

$4=3k$

$k=4/3$