

In 1-4, explain why the function has a zero in the given interval.

1  $f(x) = \frac{1}{16}x^4 - x^3 + 3; [1, 2]$

2  $f(x) = x^3 + 3x - 2; [0, 1]$

3  $f(x) = x^2 - x - \cos x; [0, \pi]$

4  $f(x) = -\frac{4}{x} + \tan\left(\frac{\pi x}{8}\right); [1, 3]$

In 5-8, verify that the Intermediate Value Theorem guarantees that there is a zero in the interval  $[0, 1]$  for the given function. Use a graphing calculator to find the zero.

5  $f(x) = x^3 + x - 1$

6  $f(x) = x^3 + 3x - 2$

7  $g(t) = 2 \cos t - 3t$

8  $h(\theta) = 1 + \theta - 3 \tan \theta$

In 9-12, verify that the Intermediate Value Theorem applies to the indicated interval and find the value of  $c$  guaranteed by the theorem. No calculator is permitted on these problems.

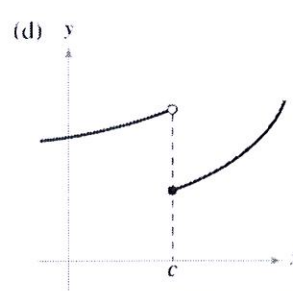
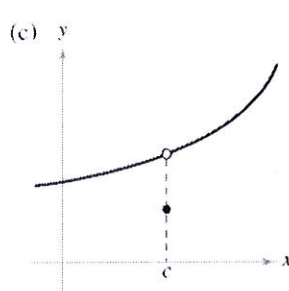
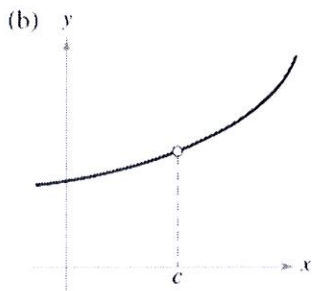
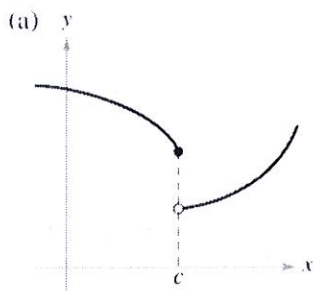
9  $f(x) = x^2 + x - 1, [0, 5], f(c) = 11$

10  $f(x) = x^2 - 6x + 8, [0, 3], f(c) = 0$

11  $f(x) = x^3 - x^2 + x - 2, [0, 3], f(c) = 4$

12  $f(x) = \frac{x^2 + x}{x - 1}, \left[\frac{5}{2}, 4\right], f(c) = 6$

13 State how continuity is destroyed at  $x = c$  for each of the graphs below:



14 Use the Intermediate Value Theorem to show that for all spheres with radii in the interval  $[1, 5]$ , there is one with a volume of 275 cubic centimeters.

15 Show that  $f(x)$  is continuous at  $x = 2$  for  $f(x) = \begin{cases} 5 - x, & -1 \leq x \leq 2 \\ x^2 - 1, & 2 < x \leq 3 \end{cases}$

16 Determine whether  $f(x)$  is continuous at  $x = -1$  for  $f(x) = \begin{cases} \frac{1}{x}, & x \leq -1 \\ \frac{x-1}{2}, & -1 < x < 1 \\ \sqrt{x}, & x \geq 1 \end{cases}$

In 1-4, explain why the function has a zero in the given interval.

1	$f(x) = \frac{1}{16}x^4 - x^3 + 3$ ; $[1, 2]$ since $f(1) = \frac{33}{16}$ (+) $f(2) = -4$ and $f(x)$ is continuous, then $f(c) = 0$
2	$f(x) = x^3 + 3x - 2$ ; $[0, 1]$ since $f(0) = -2$ + $f(1) = 2$ and $f(x)$ is continuous, then $f(c) = 0$
3	$f(x) = x^2 - x - \cos x$ ; $[0, \pi]$ since $f(0) = -1$ + $f(\pi) = \pi^2 - \pi + 1$ (+) and $f(x)$ is continuous, then $f(c) = 0$
4	$f(x) = -\frac{4}{x} + \tan\left(\frac{\pi x}{8}\right)$ ; $[1, 3]$ $f(1) = -3.586$ + $f(3) = 1.081$ by IVT, there is a zero

In 5-8, verify that the Intermediate Value Theorem guarantees that there is a zero in the interval  $[0, 1]$  for the given function. Use a graphing calculator to find the zero.

5	$f(x) = x^3 + x - 1$ $f(0) = -1$ + $f(1) = 1$ $f(.622) = 0$
6	$f(x) = x^3 + 3x - 2$ $f(0) = -2$ + $f(1) = 2$ $f(.596) = 0$
7	$g(t) = 2\cos t - 3t$ $g(0) = 2$ + $g(1) = 2\cos 1 - 3 = -1.919$ $g(.563) = 0$
8	$h(\theta) = 1 + \theta - 3\tan\theta$ $h(0) = 1$ + $h(1) =$ $h(0.45) = 0$

In 9-12, verify that the Intermediate Value Theorem applies to the indicated interval and find the value of  $c$  guaranteed by the theorem. No calculator is permitted on these problems.

9	$f(x) = x^2 + x - 1$ , $[0, 5]$ , $f(c) = 11$ $f(0) = -1$ + $f(5) = 29$ $x^2 + x - 11 = 0$ $(x+4)(x-3) = 0$ $c = 3$
10	$f(x) = x^2 - 6x + 8$ , $[0, 3]$ , $f(c) = 0$ $f(0) = 8$ + $f(3) = -1$ $x^2 - 6x + 8 = 0$ $(x-4)(x-2) = 0$ $c = 2$
11	$f(x) = x^3 - x^2 + x - 2$ , $[0, 3]$ , $f(c) = 4$ $f(0) = -2$ + $f(3) = 19$ $x^3 - x^2 + x - 2 = 4$ $x^3 - x^2 + x - 6 = 0$ $c = 2$
12	$f(x) = \frac{x^2 + x}{x - 1}$ , $\left[\frac{5}{2}, 4\right]$ , $f(c) = 6$ $f(5/2) = 5/2 = 2.5$ $f(4) = 20/3 = 6.666$ $\frac{x^2 + x}{x - 1} = 6 \rightarrow x^2 + x = 6x - 6$ $x^2 - 5x + 6 = 0$ $(x-3)(x-2) = 0$ $c = 3$

13 State how continuity is destroyed at  $x = c$  for each of the graphs below:

(a)	jump discontin. at $x=c$	(b)	remov. discontin. at $x=c$	(c)	remov. discontin. at $x=c$	(d)	jump discontin. at $x=c$
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14 Use the Intermediate Value Theorem to show that for all spheres with radii in the interval  $[1, 5]$ , there is one with a volume of 275 cubic centimeters.  $V = \frac{4}{3}\pi r^3$   $V(1) = 4.189$   $V(5) = 523.599$  so  $V(c) = 275$

15 Show that  $f(x)$  is continuous at  $x=2$  for  $f(x) = \begin{cases} 5-x, & -1 \leq x \leq 2 \\ x^2-1, & 2 < x \leq 3 \end{cases}$   $\lim_{x \rightarrow 2} f(x) = f(2)$   $3 = 5-2$   $3 = 3$  ✓

16 Determine whether  $f(x)$  is continuous at  $x=-1$  for  $f(x) = \begin{cases} \frac{1}{x}, & x \leq -1 \\ \frac{x-1}{2}, & -1 < x < 1 \\ \sqrt{x}, & x \geq 1 \end{cases}$   $\lim_{x \rightarrow -1} f(x) = f(-1)$   $-\frac{1-1}{2} = -\frac{1}{1}$   $-1 = -1$  ✓

yes