1.3 Asymptotes

Calculus Name: Key **Practice**

Identify all vertical asymptotes of each function.

1. $f(x) = \frac{x^2-x-12}{x+7}$

$x = -7$

2. $f(x) = \frac{x^3+4x^2-24x}{x^2-1x}$

$x = 1$

3. $f(x) = \frac{7x^2+4x-3}{7x-3}$

none

4. $f(x) = \frac{3x^2-11x+10}{x-2}$

none

Identify all horizontal asymptotes of each function.

5. $f(x) = \frac{\sqrt{25x^4+2x}}{x^2}$

$y = 5$

6. $f(x) = \frac{\sqrt{7x^6+3x^2+x}}{x^3+4x^2}$

$y = \sqrt{7}$

7. $f(x) = \frac{\sqrt{9x^8-2x^3-6x}}{2x^4-10x} + 3$

$y = 4\frac{1}{2}$

8. $f(x) = \frac{3x^2}{\sqrt{3x^4-2x}}$

$y = \frac{3}{\sqrt{3}} = \sqrt{3}$

Using the Squeeze Theorem, evaluate each limit. SHOW WORK!

9. $\lim_{x\to\infty} \frac{3x^6-5x^3+6}{x^3+x^8-2x^4}$

∞

10. $\lim_{x\to\infty} \cos\left(\frac{\pi x^2+\frac{\sqrt{2}}{2}x}{5-2x^2}\right)$

$\cos\left(-\frac{\pi}{2}\right) = 0$

11. $\lim_{x\to\infty} \cos\left(\frac{x^5}{e^x}\right) + 4$

$\cos(0) + 4 = 5$

Evaluate each limit.

12. $\lim_{x\to\infty} \frac{-x+2}{x^2+2x+2}$

0

13. $\lim_{x\to\infty} \left(\sin\frac{1}{x} - \frac{6x^2+2x}{3x^2}\right)$

-2

14. $\lim_{x\to\infty} \left(5\cos\frac{1}{x}\right)$

5

15. $\lim_{x\to\infty} \frac{x^7}{4^x} - 5$

-5

16. $\lim_{x\to\infty} 3^{-x} + 2$

2

17. $\lim_{x\to\infty} -3x\cos x$

DNE

18. $\lim_{x\to\infty} 2x\sin x$

DNE

19. $\lim_{x\to\infty} \frac{9x^4+4x^3+3}{x^7+2x^4+2x^3}$

0

20. $\lim_{x\to-\infty} \frac{3x^2-5x+11}{x^2-2x}$

3

21. $\lim_{x\to\infty} \cos\left(\frac{2x-\pi x^2}{x^2}\right)$

-1

22. $\lim_{x\to\infty} \left(\frac{\sin x}{x} - 4\right)$

-4

23. $\lim_{x\to\infty} \frac{-x^4-3x^2-8}{5x^4+7x+13}$

$-\frac{1}{5}$

24. $\lim_{x\to\infty} \frac{x^3-7x^2+8}{x^2+7x-2}$

∞

25. $\lim_{x\to\infty} x^2 2^{-x}$

0

26. $\lim_{x\to\infty} \frac{e^7}{9^x}$

0

27. $\lim_{x\to-\infty} \frac{3x^2-5x^7+6}{x^7-15x^4}$

-5

4. Suppose that $g(x) = \sin^2 x\sqrt{x^6+4}$, and $\lim_{x\to\infty} \frac{g(x)}{f(x)} = 0$. Which of the following functions could be f?

(A) x (B) x^2 (C) x^3 (D) x^4 (E) $\ln x$

$\lim_{x\to\infty} \frac{\sin^2 x\sqrt{x^6+\cdots}}{f(x)} = \frac{x^3}{f(x)} = 0$

↖ f(x) must be greater power than numerator in order to equal zero

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5. Which of the following statements are true for the function $f(x) = \frac{2x^3+3x+1}{2^x}$

I. $f(x)$ has a horizontal asymptote of $y = 1$ ✗
II. $f(x)$ has a horizontal asymptote of $y = 0$ ✓
III. $f(x)$ has a vertical asymptote of $x = 0$ ✗

HA: $\lim_{x\to-\infty} \frac{2x^3+\cdots}{2^x} = \frac{2(-\infty)^3}{2^{-\infty}} = 2(-\infty)^3 \cdot 2^{\infty} = -\infty$ (y=none)

$\lim_{x\to\infty} \frac{2x^3+\cdots}{2^x} = \frac{2(\infty)^3}{2^{\infty}} = 0$ (y=0)

VA: $2^x = 0$, x = none! (no VA)

(A) I only (B) II only (C) III only (D) I and III only (E) II and III only

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6. Which of the following functions has both a vertical and horizontal asymptote?

Must check each one

(A) $f(x) = \frac{1}{1+e^{-x}}$ (B) $f(x) = \tan x$ (C) $f(x) = \frac{x}{x^2+2}$

(D) $f(x) = \frac{x}{x^2-2}$ (E) $f(x) = \frac{x^2+2}{x}$

D) VA: x^2-2, $x = \pm\sqrt{2}$ VA

HA: $\lim_{x\to\infty} \frac{x}{x^2-2} = \frac{\infty}{\infty^2-2} = 0$, $y=0$ HA

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7. The function $f(x) = \begin{cases} \frac{x^2+2x+3}{x^2-1}, & x \geq 0 \\ \frac{x}{e^x}, & x < 0 \end{cases}$ has which of the following asymptotes?

VA: $x^2-1=0$, $x = \pm 1$

$e^x = 0$, x = none

HA: $\lim_{x\to-\infty} \frac{x}{e^x} = \frac{-\infty}{e^{-\infty}} = -\infty \cdot e^{\infty} = -\infty$ (y=none)

$\lim_{x\to\infty} \frac{x^2+\cdots}{x^2+\cdots} = x^2/x^2 = 1$, $y=1$

(A) $y = 0$ only. (B) $y = 1$ only. (C) $y = 1$, $x = 1$ only.

(D) $y = 1$, $x = \pm 1$ only. (E) $y = 0$, $y = 1$, $x = \pm 1$.

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8. If the function $f(x) = \frac{-ax^3+bx^2+cx+d}{e^{-x}-wx^3+w}$ has a horizontal asymptote of $y = 2$ and a vertical asymptote of $x = 0$, then $w - a =$

(A) -1 (B) 0 (C) 1 (D) ∞ (E) The limit does not exist.

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9. What are all horizontal asymptotes of the graph of $y = \frac{5+2^x}{1-2^x}$ in the xy-plane?

(A) $y = -1$ only (B) $y = 0$ only (C) $y = 5$ only

(D) $y = -1$ and $y = 0$ (E) $y = -1$ and $y = 5$

$\lim_{x\to-\infty} \frac{5+2^{-\infty}}{1-2^{-\infty}} = \frac{5+\frac{1}{2^{\infty}}}{1-\frac{1}{2^{\infty}}} = \frac{5+0}{1-0} = 5$

$\lim_{x\to\infty} \frac{5+2^{\infty}}{1-2^{\infty}} = \frac{5+\infty}{1-\infty} = \frac{\infty}{-\infty} = -1$

HA: $y = -1$