Unit 1 Test Review

AP Calculus BC

Name KEY

____1. $\lim_{x\to 3}\frac{x^2-9}{x^2+2x-15}$

a. $-\infty$ b. 0 c. $\frac{3}{4}$ d. ∞ e. DNE

____2. $\lim_{x\to -5}\frac{x^2-9}{x^2+2x-15}$

a. $-\infty$ b. 0 c. $\frac{3}{4}$ d. ∞ e. DNE

____3. $\lim_{x\to \infty}\frac{x^2-9}{x^2+2x-15}$

a. $-\infty$ b. -1 c. $\frac{3}{4}$ d. 1 e. ∞

____4. $\lim_{x\to -\infty}\frac{x^2-9}{x^2+2x-15}$

a. $-\infty$ b. -1 c. $\frac{3}{4}$ d. 1 e. ∞

____5. $\lim_{x\to 0} 1-e^{-x}$

a. $-\infty$ b. 0 c. 1 d. ∞ e. DNE

____6. $\lim_{x\to -\infty} 1-e^{-x}$

a. $-\infty$ b. 0 c. 1 d. e e. ∞

____7. $\lim_{x\to \infty} 1-e^{-x}$

a. $-\infty$ b. 0 c. 1 d. ∞ e. DNE

____8. $\lim_{x\to \infty}\frac{3x}{2x^2+7}$

a. $-\infty$ b. $-\frac{3}{2}$ c. 0 d. $\frac{3}{2}$ e. ∞

____9. $\lim_{x\to -\infty}\frac{4x^3}{2x^4+1}$

a. $-\infty$ b. -2 c. 0 d. 2 e. ∞

____10. $\lim_{x\to \infty}\frac{16x^3-2x+5}{-8x^3+7}$

a. $-\infty$ b. -2 c. 0 d. 2 e. ∞

____11. $\lim_{x\to 1}\frac{x^4-1}{x-1}$

a. 0 b. 1 c. 2 d. 4 e. DNE

____12. $\lim_{x\to -3}\frac{x^2+x-6}{x^2-9}$

a. $\frac{-5}{6}$ b. $\frac{-2}{3}$ c. $\frac{2}{3}$ d. $\frac{5}{6}$ e. DNE

____13. $\lim_{x\to 4}\frac{\sqrt{x}-2}{x-4}$

a. $\frac{-1}{4}$ b. 0 c. $\frac{1}{4}$ d. $\frac{1}{2}$ e. DNE

____14. $\lim_{x\to\infty}\frac{1-x}{\sqrt{3x^2-x}}$

a. $\frac{-1}{3}$ b. $\frac{-1}{\sqrt{3}}$ c. $\frac{1}{\sqrt{3}}$ d. $\frac{1}{3}$ e. DNE

____15. $\lim_{x\to 2}\frac{\frac{x+2}{x}-2}{x-2}$

a. -2 b. $-1/2$ c. $1/2$ d. 2 e. DNE

____16. $\lim_{x\to 2}\left(\frac{1}{4x-8}-\frac{1}{x^2-4}\right)$

a. $-\infty$ b. -3 c. $1/16$ d. ∞ e. DNE

____17. $\lim_{x\to 6}\frac{x-\sqrt{5x+6}}{x-6}$

a. -5 b. $\frac{7}{12}$ c. 0 d. ∞ e. DNE

____18. $\lim_{x\to 0}\frac{(3+x)^2-9}{x}$

a. 3 b. 6 c. 9 d. 0 e. DNE

____19. $\lim_{x\to 0} e^{-x^2}\cos(x)$

a. -1 b. 0 c. 1 d. ∞ e. DNE

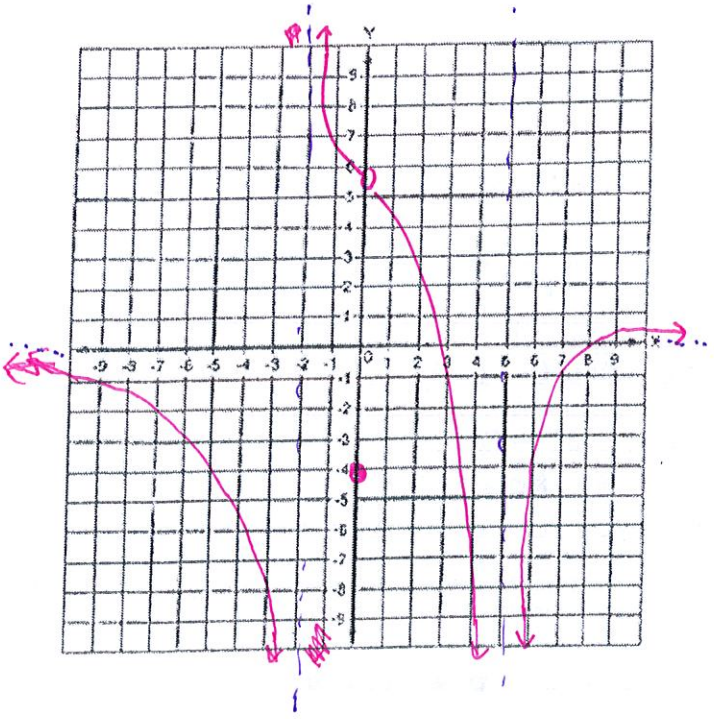
____20. $\lim_{x\to\infty} e^{-x^2}\cos(x)$

a. -1 b. 0 c. 1 d. ∞ e. DNE

Sketch a graph with the following characteristics:

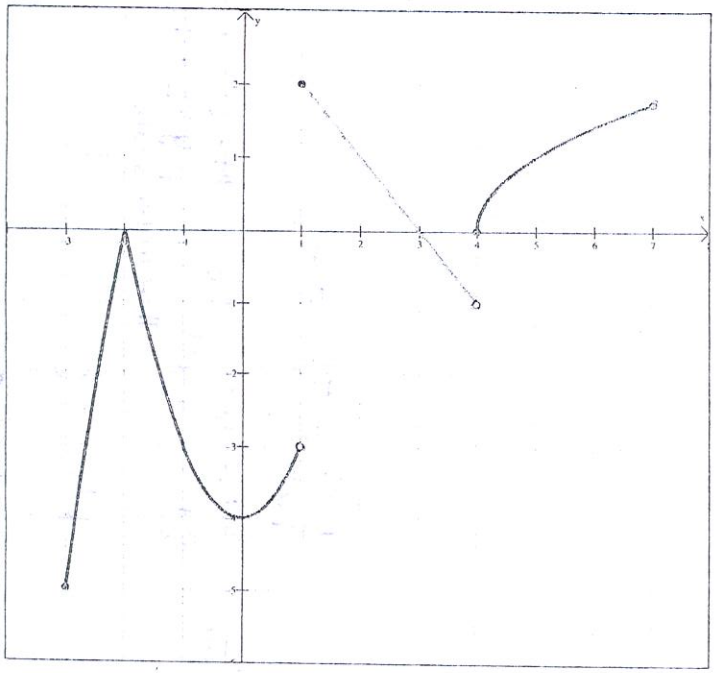
$\lim_{x\to -2^-} f(x)=-\infty$; $\lim_{x\to -2^+} f(x)=\infty$; $\lim_{x\to 5} f(x)=-\infty$

$\lim_{x\to -\infty} f(x)=0$; $\lim_{x\to\infty} f(x)=0$; $f(0)=-4$



22. Limits and Continuity

refer to the graph of $f(x)$ below.



a) $\lim_{x \to -1^-} f(x)$ -3

b) $\lim_{x \to -1^+} f(x)$ -3

c) $\lim_{x \to -1} f(x)$ -3

d) $f(-1)$ -3

e) $\lim_{x \to 1^-} f(x)$ -3

f) $\lim_{x \to 1^+} f(x)$ 2

g) $\lim_{x \to 1} f(x)$ DNE

h) $f(1)$ 2

i) $\lim_{x \to 4^-} f(x)$ -1

j) $\lim_{x \to 4^+} f(x)$ 0

k) $\lim_{x \to 4} f(x)$ DNE

l) $f(4)$ 0

Limits and Continuity

For #11-12, determine whether the given function is continuous as the specified value of x. Remember to use ALL three tests to justify your answer.

23. Determine whether $f(x)$ is continuous at x = -1.

$$f(x) = \begin{cases} \dfrac{x^2 + x}{x + 1} & x < -1 \\ \dfrac{x - 4}{x - 1} & x \geq -1 \end{cases}$$

jump discont.

① $f(-1) = \frac{-5}{-2} = 5/2$

② $\lim_{x \to -1} f(x) =$ DNE

③ not cont: $f(-1) \neq \lim_{x \to -1} f(x)$

24. Determine whether $f(x)$ is continuous at x = 6.

$$f(x) = \begin{cases} x^2 - 7 & 2 \leq x < 6 \\ 29 & x = 6 \\ 3x + 11 & x > 6 \end{cases}$$

① $f(6) = 29$

② $\lim_{x \to 6} f(x) = 29$

③ yes cont: $f(6) = \lim_{x \to 6} f(x)$

25. Determine whether the functions below are continuous or discontinuous. If there exists a discontinuity at any point, identify the discontinuity as removable or non-removable.

(a) $f(x) = \dfrac{x^2 - 9}{x^2 - 7x - 30}$

$f(x) = \dfrac{(x+3)(x-3)}{(x-10)(x+3)}$

Discont. @

x = -3 (removable hole)

x = 10 (non-removable VA)

(b) $f(x) = \dfrac{2x + 1}{x^2 + 6x + 5}$

$f(x) = \dfrac{2x+1}{(x+5)(x+1)}$

Discont. @

x = -5 (non-remov. VA)

x = -1 (non-remov. VA)

Average ROC vs. Instantaneous ROC

26. $f(x) = 3x^2 - x + 1$

a. Find the average ROC over the interval [0, 3]

$\frac{f(3)-f(0)}{3-0} = \frac{25-1}{3} = \frac{24}{3} = 8$

b. Find the equation of the secant line over the interval [0, 3]

pt: ★(3, 25) OR (0, 1)★

$y - 25 = 8(x-3)$ OR $y - 1 = 8(x-0)$

THESE ARE THE SAME

c. Find the instantaneous ROC at x = 3

$\lim_{h\to 0} \frac{f(3+h)-f(3)}{h} = \frac{3(3+h)^2 - (3+h) + 1 - 7}{h}$

$= \frac{27 + 18h + 3h^2 - 3 - h + 1 - 28}{h} = \frac{h(18+3h-1)}{h}$

$= 18 + 3h - 1 = 17 + 3h = 17$

d. Find the equation of the tangent line at x = 3

pt: (3, 25)

$y - 25 = 17(x-3)$

27. $h(0) = 1,\ h(3) = -100,\ h'(3) = -4$

a. Find the slope of the secant line over the interval [0, 3]

$\frac{h(3)-h(0)}{3-0} = \frac{-100-1}{3-0} = \frac{-101}{3}$

b. Find the equation of the secant line over the interval [0, 3]

pt: ★(0, 1) OR ★(3, -100)

$y - 1 = \frac{-101}{3}(x-0)$ OR $y + 100 = \frac{-101}{3}(x-3)$

THESE are the SAME

c. Find the derivative at x = 3

easy! $h'(3) = -4$

d. Find the equation of the tangent line at x = 3

pt: (3, -100) $y + 100 = -4(x-3)$

e. Is there a value of x s.t. h(x) = - 5?

If h(x) is continuous over [0, 3], then by IVT, there is an x s.t. h(x) = -5.

28. $g(x) = \frac{1}{x+4}$

a. Find the average slope over the interval [0, 3]

$\frac{g(3)-g(0)}{3-0} = \frac{\frac{1}{7} - \frac{1}{4}}{3} = \frac{\frac{-3}{28}}{3} = \frac{-1}{28}$

b. Find the equation of the secant line over the interval [0, 3]

pt: ★(0, 1/4) OR pt: ★(3, 1/7)

$y - \frac{1}{4} = \frac{-1}{28}(x-0)$ OR $y - \frac{1}{7} = \frac{-1}{28}(x-3)$

c. Find g'(3) THESE ARE THE SAME

$\lim_{h\to 0} \frac{g(3+h)-g(3)}{h} = \frac{\frac{1}{3+h+4} - \frac{1}{7}}{h} = \frac{\frac{1}{7+h} - \frac{1}{7}}{h}$

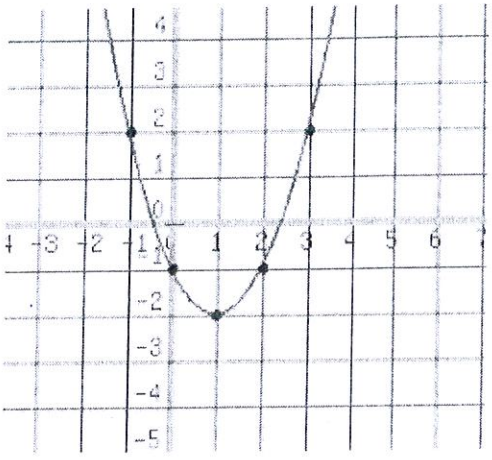
$= \frac{7 - 7 - h}{7(7+h)} \cdot \frac{1}{h} = \frac{-h}{7h(7+h)} = \frac{-1}{7(7+h)} = \frac{-1}{49}$

d. Find the equation of the tangent line at x = 3

pt: (3, 1/7)

$y - \frac{1}{7} = \frac{-1}{49}(x-3)$

29. k(x) =



a. Find the average slope over the interval [0, 1]

$\frac{\text{rise}}{\text{run}} = \frac{-1}{1} = -1$

b. Find the equation of the secant line over the interval [0, 1]

pt: (0, -1)★ OR pt: (1, -2)★

$y + 1 = -1(x-0)$ $y + 2 = -1(x-1)$

These are the SAME

c. Find k'(1)

$k'(1) = 0$

d. Find the equation of the tangent line at x = 1

pt: (1, -2)

$y + 2 = 0(x-1)$