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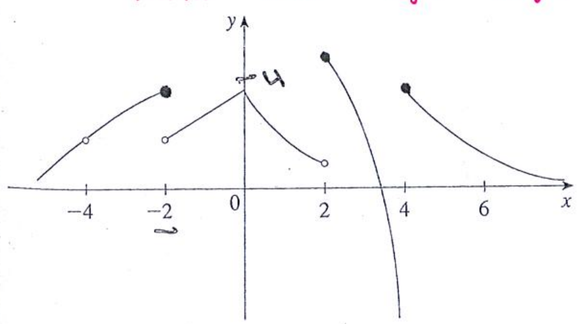
**1.** Write an equation that expresses the fact that a function $f$ is continuous at the number 4. $f(4) = \lim_{x \to 4} f(x)$

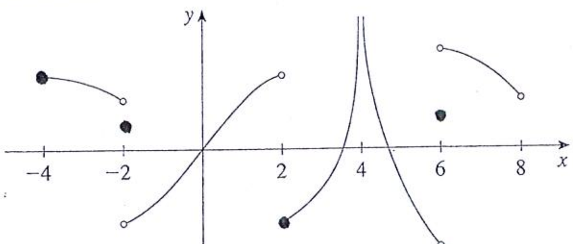
**2.** If $f$ is continuous on $(-\infty, \infty)$, what can you say about its graph? - defined everywhere
- no discontinuities!

**3.** (a) From the graph of $f$, state the numbers at which $f$ is discontinuous and explain why. $x=-4$ removable, $x=-2$ jump, $x=2$ jump, $x=4$ infinite + jump

(b) For each of the numbers stated in part (a), determine whether $f$ is continuous from the right, or from the left, or neither. $x=-4$ neither, $x=-2$ left, $x=2$ right, $x=4$ right.




**4.** From the graph of $g$, state the intervals on which $g$ is continuous.




$[-4,-2) \cup (-2,2) \cup (2,4) \cup (4,6) \cup (6,8)$

**5–8** Sketch the graph of a function $f$ that is continuous except for the stated discontinuity.

answers vary!

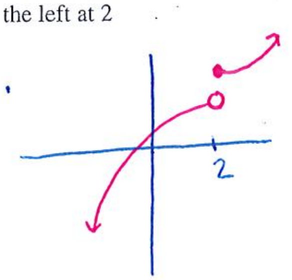
**5.** Discontinuous, but continuous from the right, at 2

**6.** Discontinuities at $-1$ and 4, but continuous from the left at $-1$ and from the right at 4

**7.** Removable discontinuity at 3, jump discontinuity at 5

**8.** Neither left nor right continuous at $-2$, continuous only from the left at 2

5.

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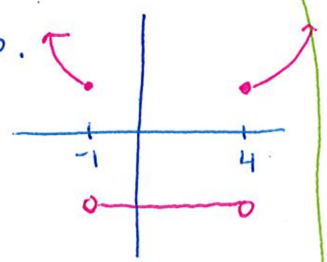


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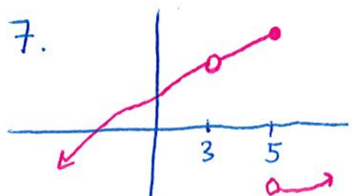


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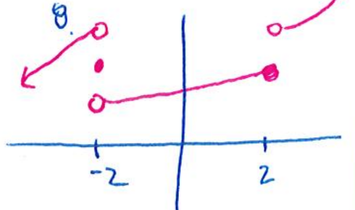


**9.** The toll $T$ charged for driving on a certain stretch of a toll road is \$5 except during rush hours (between 7 AM and 10 AM and between 4 PM and 7 PM) when the toll is \$7. See below

(a) Sketch a graph of $T$ as a function of the time $t$, measured in hours past midnight.

(b) Discuss the discontinuities of this function and their significance to someone who uses the road.

**10.** Explain why each function is continuous or discontinuous.

(a) The temperature at a specific location as a function of time continuous

(b) The temperature at a specific time as a function of the distance due west from New York City continuous

(c) The altitude above sea level as a function of the distance due west from New York City discontinuous (think cliff!)

(d) The cost of a taxi ride as a function of the distance traveled discont. (price jump)

(e) The current in the circuit for the lights in a room as a function of time discontinuous (lights switched on and off)

**11.** Suppose $f$ and $g$ are continuous functions such that $g(2) = 6$ and $\lim_{x \to 2} [3f(x) + f(x)g(x)] = 36$. Find $f(2)$.

see below

**12–14** Use the definition of continuity and the properties of limits to show that the function is continuous at the given number $a$.

**12.** $f(x) = 3x^4 - 5x + \sqrt[3]{x^2 + 4}, \quad a = 2$

**13.** $f(x) = (x + 2x^3)^4, \quad a = -1$

**14.** $h(t) = \dfrac{2t - 3t^2}{1 + t^3}, \quad a = 1$

work below

9a


9b. - jump discontinuities at hours 7:00, 10:00 am & 4:00, 7:00pm
- this is important someone on the toll road because at these times, the prices increase or decrease \$2!

11. given: $g(2) = 6$ + $\lim_{x \to 2} 3f(x) + f(x)g(x) = 36$

try substitution: $\lim_{x \to 2} 3f(2) + f(2)g(2) = 36$

$3f(2) + f(2) \cdot 6 = 36$

$9f(2) = 36$

$f(2) = 4$

12. $\boxed{\lim_{x\to 2} f(x) = f(2)}$ → def. of continuity

$$\lim_{x\to 2} 3x^4 - 5x + \sqrt[3]{x^2+4}$$
$$= 3(2)^4 - 5(2) + \sqrt[3]{2^2+4}$$
$$= 3(16) - 10 + \sqrt[3]{8}$$
$$= 48 - 10 + 2$$
$$= 38 + 2$$
$$= 40$$

$$f(2) = 3(2)^4 - 5(2) + \sqrt[3]{2^2+4}$$
$$= 3(16) - 10 + \sqrt[3]{8}$$
$$= 48 - 10 + 2$$
$$= 38 + 2$$
$$= 40$$

=

We can see limit value equals function value so f(x) IS cont. at x=2

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13. $\boxed{\lim_{x\to -1} f(x) = f(-1)}$ → def. of contin.

$$\lim_{x\to -1} (-1 + 2(-1)^3)^4$$
$$= (-1-2)^4$$
$$= (-3)^4$$
$$= 243$$

$$f(-1) = (-1 + 2(-1)^3)^4$$
$$= (-1-2)^4$$
$$= (-3)^4$$
$$= 243$$

=

We can see limit value equals function value so f(x) IS cont. at x=-1

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14. $\boxed{\lim_{t\to 1} h(t) = h(1)}$ → def. of continuity

$$\lim_{t\to 1} \frac{2(1) - 3(1)^2}{1 + (1)^3}$$
$$= \frac{2-3}{2}$$
$$= -\frac{1}{2}$$

$$h(1) = \frac{2(1) - 3(1)^2}{1 + (1)^3}$$
$$= \frac{2-3}{2}$$
$$= -\frac{1}{2}$$

=

limit value equals function value so h(t) IS continuous at x=1