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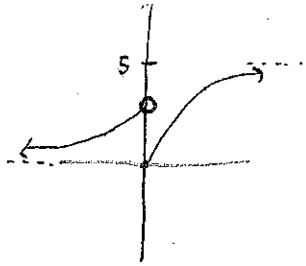
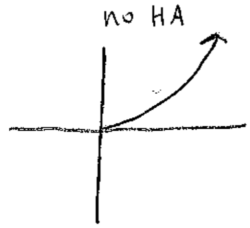
True / False

11) True:



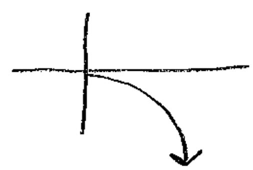
or $f(x) = \begin{cases} \frac{5x}{x+1}, & x > 0 \\ e^x, & x < 0 \end{cases}$

12) True:



$\lim_{x \to \infty} f(x) = \infty$

or



$\lim_{x \to \infty} f(x) = -\infty$

13) True: $y = \frac{4}{x-1} \rightarrow x = 1$ is a VA & $y(1) =$ DNE (is not defined)

(99% of cases TRUE; look at graph p.167, #1 at x = 2 for 1% case of false)

14) False: might be true but only if we know $f(x)$ is continuous on $[1, 3]$ (which we don't know)

15) True: By definition, if f is cont. at 5 then

$$\left[\lim_{x \to 5} f(x) = f(5)\right]$$

$$\lim_{x \to 2} f(4(2)^2 - 11) = 2$$

$$\lim_{x \to 2} f(5) = 2$$

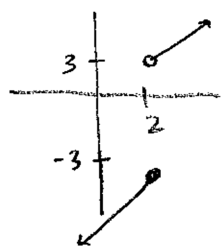
$$\downarrow$$

$$\lim_{x \to 2} 2 = 2$$

22) False: $y(0) = 5$, $y(2) = 989$ } although y is continuous, we can't say for sure there is a root $(y(c) = 0)$ since 0 is not in the interval $(5, 989)$

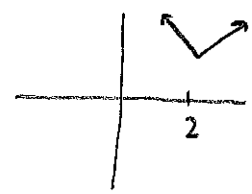
23) True

24) False: $f \rightarrow$



not cont.

$|f| \rightarrow$



cont.

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Exercises

1) a. (i) 3 (ii) 0

(iii) DNE (iv) 2

(v) ∞ (vi) $-\infty$

(vii) 4 (vii) -1

b. HA: $y = 4$, $y = -1$

c. VA: $x = 0$, $x = 2$

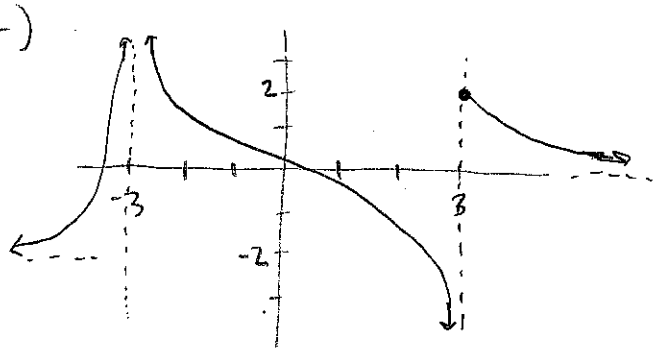
d. $x = -3$: jump

$x = 0$: infinite

$x = 2$: infinite, jump

$x = 4$: removable

2)



3) $\lim_{x\to 1} e^{1^3-1} = e^{1-1} = e^0 = 1$

4) $\lim_{x\to 3} \frac{(3)^2-9}{(3)^2+2(3)-3} = \frac{9-9}{12} = \frac{0}{12} = 0$

5) $\lim_{x\to -3} \frac{(x-3)\cancel{(x+3)}}{\cancel{(x+3)}(x-1)} = \frac{x-3}{x-1} = \frac{-3-3}{-3-1} = \frac{-6}{-4} = \frac{3}{2}$

6) $\lim_{x\to 1^+} \frac{(x-3)\cancel{(x+3)}}{\cancel{(x+3)}(x-1)} = \frac{x-3}{x-1}$ [PAUSE! from this step we see $x=1$ is a VA! By def, $\lim_{x\to 1^+} f(x) = \infty$ OR $-\infty$.] $= -\infty$

7) $\lim_{h\to 0} \frac{(h-1)^3+1}{h} = \frac{(h-1)(h^2-2h+1)+1}{h} = \frac{\cancel{h^3}-2h^2+h-\cancel{h^3}+2h-\cancel{1}+\cancel{1}}{h}$

$= \frac{-2h^2+3h}{h} = \frac{\cancel{h}(-2h+3)}{\cancel{h}} = -2(0)+3 = 3$

8) $\lim_{t\to 2} \frac{(t-\cancel{2})(t+2)}{(t\cancel{-2})(t^2+2t+4)} = \frac{t+2}{t^2+2t+4} = \frac{2+2}{2^2+2(2)+4} = \frac{4}{12} = \frac{1}{3}$

12) $\lim_{x\to 3} \frac{\sqrt{x+6}-x}{x^3-3x^2} \cdot \frac{\sqrt{x+6}+x}{\sqrt{x+6}+x} = \frac{x+6-x^2}{(x^3-3x^2)(\sqrt{x+6}+x)} = \frac{-1(x^2-x-6)}{x^3(x-3)(\sqrt{x+6}+x)}$

$= \frac{-1\cancel{(x-3)}(x+2)}{x^2\cancel{(x-3)}(\sqrt{x+6}+x)} = \frac{-1(x+2)}{x^2(\sqrt{x+6}+x)} = \frac{-(3+2)}{(3)^2(\sqrt{3+6}+3)} = \frac{-5}{54}$

13) $\lim_{x\to\infty} \frac{\sqrt{x^2\cancel{-9}}}{2x\cancel{-6}} = \frac{\sqrt{x^2}}{2x} = \frac{\cancel{x}}{2\cancel{x}} = \frac{1}{2}$

14) $\lim_{x\to-\infty} \frac{\sqrt{x^2\cancel{-9}}}{2x\cancel{-6}} = \frac{\sqrt{x^2}}{2x} = \frac{\sqrt{(-\infty)^2}}{2(-\infty)} = \frac{\sqrt{\infty^2}}{-2\infty} = \frac{\infty}{-2\infty} = -\frac{1}{2}$

16) $\lim_{x\to-\infty} \frac{\cancel{1+2x^7}-x^4}{\cancel{5+x}-3x^4} = \frac{-\cancel{x^4}}{-3\cancel{x^4}} = \frac{1}{3}$

17) $\lim_{x\to\infty} \sqrt{x^2\cancel{+4x+1}} - x = \sqrt{x^2} - x = x - x = 0$

18) $\lim_{x\to\infty} e^{x-x^2} = e^{\infty-\infty^2} = e^{-\infty} = \frac{1}{e^{\infty}} = 0$

20) $\lim_{x\to 1} \left(\frac{1}{x-1}\cdot\frac{x-2}{x-2} + \frac{1}{(x-2)(x-1)}\right) = \frac{x-2+1}{(x-2)(x-1)} = \frac{\cancel{x-1}}{\cancel{(x-1)}(x-2)} = \frac{1}{x-2} = \frac{1}{1-2} = -1$

29) a. (i) 3 (ii) 0 (iii) DNE

(iv) 0 (v) 0 (vi) 0

b. $x=0$; $x=3$

jump | removable