purple is unsimplified answer

red is simplified answer

blue is original problem rewritten

Calculus Worksheet
Chain Rule Practice #1

Find the derivative of each of the following functions.

1. $y = (x^2 + 4x + 6)^5$

$y' = 5(x^2+4x+6)^4 \cdot (2x+4)$

$y' = 5(2x+4)(x^2+4x+6)^4$

2. $y = \tan 3x$

$\frac{dy}{dx} = \sec^2(3x) \cdot 3$

$\frac{dy}{dx} = 3\sec^2(3x)$

3. $f(x) = (x^3 - 5x)^4$

$f'(x) = 4(x^3-5x)^3 \cdot (3x^2-5)$

$f'(x) = 4(3x^2-5)(x^3-5x)^3$

4. $y = 4\sec 5x$

$y' = 4\tan(5x)\sec(5x) \cdot 5$

$y' = 20\tan(5x)\sec(5x)$

5. $f(x) = (3x-2)^{10}(5x^2 - x + 1)^{12}$

$f' = \underbrace{(5x^2-x+1)^{12}}_{g} \cdot \underbrace{10(3x-2)^9 \cdot 3}_{f'}$

$+ \underbrace{(3x-2)^{10}}_{f} \cdot \underbrace{12(5x^2-x+1)^{11} \cdot (10x-1)}_{g'}$

$f' = 30(5x^2-x+1)^{12}(3x-2)^9 + 12(3x-2)^{10}(5x^2-x+1)^{11}(10x-1)$

6. $y = \cos(x^3)$

$y' = -\sin(x^3) \cdot 3x^2$

$y' = -3x^2\sin(x^3)$

7. $f(x) = (6x^2+5)^3(x^3-7)^4$

$\frac{dy}{dx} = \underbrace{(x^3-7)^4}_{g} \cdot \underbrace{3(6x^2+5)^2 \cdot 12x}_{f'}$

$+ \underbrace{(6x^2+5)^3}_{f} \cdot \underbrace{4(x^3-7)^3 \cdot 3x^2}_{g'}$

8. $y = \cos^3 x = (\cos x)^3$

$\frac{dy}{dx} = 3\cos^2 x \cdot -\sin x$

$\frac{dy}{dx} = -3\sin x \cos^2 x$

9. $y = (2x^2 - 6x + 1)^{-8}$

$\frac{dy}{dx} = -8(2x^2-6x+1)^{-9} \cdot (4x-6)$

$\frac{dy}{dx} = -8(4x-6)(2x^2-6x+1)^{-9}$

10. $f(x) = (1+\cos^2 x)^6 = (1+(\cos x)^2)^6$

$\frac{dy}{dx} = 6(1+\cos^2 x)^5 \cdot (2\cos x \cdot -\sin x)$

$\frac{dy}{dx} = -6(2\cos x \cdot \sin x)(1+\cos^2 x)^5$

purple is unsimplified answer

red is simplified answer

blue is original problem rewritten

Calculus Worksheet
Chain Rule Practice #2

Find the derivative of each of the following functions.

1. $y=\sqrt{x^2-7x}$ $= (x^2-7x)^{1/2}$

$y' = \frac{1}{2}(x^2-7x)^{-1/2}\cdot(2x-7)$

$y' = \frac{2x-7}{2\sqrt{x^2-7x}}$

2. $y=\tan(x^2)+\tan^2 x$ $= \tan(x^2)+(\tan x)^2$

$y' = \sec^2(x^2)\cdot 2x + 2\tan x\cdot\sec^2 x$

$y' = 2x\sec^2(x^2) + 2\tan x\sec^2 x$

3. $y=\frac{1}{(x^2-2x-5)^4}$ $= (x^2-2x-5)^{-4}$

$y' = -4(x^2-2x-5)^{-5}\cdot(2x-2)$

$y' = \frac{-4(2x-2)}{(x^2-2x-5)^5}$

$= \frac{-8x+8}{(x^2-2x-5)^5}$

4. $y=\cos(\tan x)$

$\frac{dy}{dx} = -\sin(\tan x)\cdot\sec^2 x$

$\frac{dy}{dx} = -\sin(\tan x)\sec^2 x$

5. $f(x)=\left(x-\frac{1}{x}\right)^{\frac{1}{2}}$ $= (x-x^{-1})^{1/2}$

$f' = \frac{1}{2}(x-x^{-1})^{-1/2}\cdot(1+x^{-2})$

$f' = \frac{1}{2\sqrt{x-\frac{1}{x}}}\cdot(1+x^{-2})$

6. $y=\sin^3 x+\cos^3 x$ $= (\sin x)^3+(\cos x)^3$

$y' = 3\sin^2 x\cdot\cos x + 3\cos^2 x\cdot(-\sin x)$

$y' = 3\sin^2 x\cos x - 3\cos^2 x\sin x$

7. $y=\left(\frac{x-6}{x+7}\right)^3$

$\frac{dy}{dx} = 3\left(\frac{x-6}{x+7}\right)^2\cdot\left(\frac{(x+7)(1)-(x-6)(1)}{(x+7)^2}\right)$

$\frac{dy}{dx} = 3\left(\frac{x-6}{x+7}\right)^2\cdot\frac{13}{(x+7)^2} = \frac{39(x-6)^2}{(x+7)^4}$

8. $y=\sin^2(\cos(4x))$ $= (\sin(\cos(4x)))^2$

$y' = 2(\sin(\cos 4x))\cdot\cos(\cos 4x)\cdot(-\sin 4x)\cdot 4$

$y' = -8\sin(\cos 4x)\cos(\cos 4x)\sin 4x$

9. $y=\frac{1}{\sqrt[5]{2x-1}}$ $= (2x-1)^{-1/5}$

$\frac{dy}{dx} = -\frac{1}{5}(2x-1)^{-6/5}\cdot 2$

$\frac{dy}{dx} = -\frac{2}{5}(2x-1)^{-6/5}$

10. $y=\frac{\sin^2 x}{\cos x}$ $= \sin x\tan x$

$y' = \tan x\cos x + \sin x\sec^2 x$

$y' = \tan x\cos x + \sin x\sec^2 x$

11. $y=\sin^3(2x+3)$ $= (\sin(2x+3))^3$

$y' = 3(\sin(2x+3))^2\cdot\cos(2x+3)\cdot 2$

$y' = 6\sin^2(2x+3)\cos(2x+3)$