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41. If $f(x) = x^2/(1 + x)$, find $f''(1)$.

[redacted]

43. Suppose that $f(5) = 1$, $f'(5) = 6$, $g(5) = -3$, and $g'(5) = 2$. Find the following values.

(a) $(fg)'(5)$ (b) $(f/g)'(5)$ (c) $(g/f)'(5)$

44. Suppose that $f(2) = -3$, $g(2) = 4$, $f'(2) = -2$, and $g'(2) = 7$. Find $h'(2)$.

(a) $h(x) = 5f(x) - 4g(x)$ (b) $h(x) = f(x)g(x)$

(c) $h(x) = \dfrac{f(x)}{g(x)}$ (d) $h(x) = \dfrac{g(x)}{1 + f(x)}$

[redacted]

46. If $h(2) = 4$ and $h'(2) = -3$, find

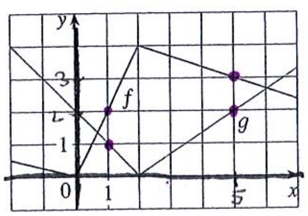
$$\left.\frac{d}{dx}\left(\frac{h(x)}{x}\right)\right|_{x=2}$$

47. If $g(x) = xf(x)$, where $f(3) = 4$ and $f'(3) = -2$, [redacted] Find $g'(3)$.

48. If $f(2) = 10$ and $f'(x) = x^2 f(x)$ for all x, find $f''(2)$.

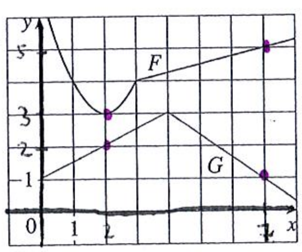
49. If f and g are the functions whose graphs are shown, let $u(x) = f(x)g(x)$ and $v(x) = f(x)/g(x)$.

(a) Find $u'(1)$. (b) Find $v'(5)$.



50. Let $P(x) = F(x)G(x)$ and $Q(x) = F(x)/G(x)$, where F and G are the functions whose graphs are shown.

(a) Find $P'(2)$. (b) Find $Q'(7)$.



(41) $f(x) = \dfrac{x^2}{(1+x)}$]f]g

HOW TO: use quotient rule TWICE + then plug in $x=1$

$$f'(x) = \frac{(1+x)(2x) - (x^2)(1)}{(1+x)^2} = \frac{2x + 2x^2 - x^2}{(1+2x+x^2)}$$

$$= \frac{x^2 + 2x}{(1+2x+x^2)}$$]f]g

* note: although simplifying is not necessary, because we have to take 2nd derivative, it's <u>so</u> much easier to simplify the 1st derivative first

$$f''(x) = \frac{(1+2x+x^2)(2x+2) - (x^2+2x)(2+2x)}{(1+2x+x^2)^2}$$

$$f''(1) = \frac{(1+2(1)+(1)^2)(2(1)+2) - ((1)^2+2(1))(2+2(1))}{(1+2(1)+(1)^2)^2}$$

$$= \frac{(4)(4) - (3)(4)}{(4)^2} = \frac{16-12}{16} = \frac{4}{16} = \boxed{\frac{1}{4}}$$

---

(43) Suppose $f(5)=1$, $f'(5)=6$, $g(5)=-3$, $g'(5)=2$.

a. $(fg)'(5)$

$= g \cdot f' + fg'$

$= g(5)f'(5) + f(5)g'(5)$

$= (-3)(6) + (1)(2)$

$= -18 + 2 = \boxed{-16}$

b. $(f/g)'(5)$

$$= \frac{gf' - fg'}{g^2}$$

$$= \frac{g(5)f'(5) - f(5)g'(5)}{[g(5)]^2}$$

$$= \frac{(-3)(6) - (1)(2)}{(-3)^2}$$

$$= \frac{-18-2}{9} = \boxed{\frac{-20}{9}}$$

c. $(g/f)'(5)$

$$= \frac{f \cdot g' - g \cdot f'}{f^2}$$

$$= \frac{f(5)\,g'(5) - g(5)\,f'(5)}{[f(5)]^2}$$

$$= \frac{(1)(2) - (-3)(6)}{(1)^2} = \frac{2+18}{1} = \boxed{20}$$

---

(44) Suppose that $f(2) = -3$, $g(2) = 4$, $f'(2) = -2$, $g'(2) = 7$.

Find $h'(2)$.

a. $h(x) = 5f(x) - 4g(x)$

$h'(x) = 5f'(x) - 4g'(x)$

$h'(2) = 5f'(2) - 4g'(2)$

$= (5)\cdot(-2) - (4)\cdot(7)$

$= -10 - 28 = \boxed{-38}$

b. $h(x) = f(x)g(x)$

$h'(x) = g(x)f'(x) + f(x)g'(x)$

$h'(2) = g(2)f'(2) + f(2)g'(2)$

$= (4)\cdot(-2) + (-3)\cdot(7)$

$= -8 - 21 = \boxed{-29}$

c. $h(x) = \frac{f(x)}{g(x)}$

$$h'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

$$h'(2) = \frac{g(2)f'(2) - f(2)g'(2)}{[g(2)]^2}$$

$$= \frac{(4)(-2) - (-3)(7)}{(4)^2}$$

$$= \frac{-8+21}{16} = \boxed{\frac{13}{16}}$$

d. $h(x) = \frac{g(x)}{1+f(x)}$

$$h'(x) = \frac{(1+f(x))\,g'(x) - g(x)\,(f'(x))}{(1+f(x))^2}$$

$$h'(2) = \frac{(1+f(2))\,g'(2) - g(2)\,f'(2)}{(1+f(2))^2}$$

$$= \frac{(1+-3)(7) - (4)(-2)}{(1+-3)^2}$$

$$= \frac{(-2)(7) - (4)(-2)}{(-2)^2}$$

$$= \frac{-14+8}{4} = \frac{-6}{4} = \boxed{\frac{-3}{2}}$$

(46) $h(2) = 4$ and $h'(2) = -3$

Find $\frac{d}{dx}\left(\frac{h(x)}{x}\right)\Big|_{x=2}$

HOW TO: quotient rule + then plug in $x=2$

$$= \frac{(x)\,h'(x) - h(x)\cdot 1}{x^2}$$

$$= \frac{(2)\,h'(2) - h(2)\cdot 1}{(2)^2} = \frac{(2)(-3) - (4)(1)}{4} = \frac{-6-4}{4} = \boxed{\frac{-5}{2}}$$

---

(47) If $g(x) = x f(x)$, $f(3) = 4$ + $f'(3) = -2$,

find $g'(3)$

$$g'(x) = f(x)\cdot(1) + (x)\,f'(x)$$

$$g'(3) = f(3)\cdot 1 + (3)\cdot f'(3)$$

$$= (4)(1) + (3)(-2)$$

$$= 4 - 6 = \boxed{-2}$$

---

(48) If $f(2) = 10$ + $f'(x) = x^2 f(x)$,

find $f''(2)$

$$f''(x) = f(x)\cdot 2x + (x^2)\,f'(x)$$

$$f''(2) = f(2)\cdot(2\cdot 2) + (2^2)\,f'(2)$$

$$= (10)(4) + (4)\left(2^2\cdot f(2)\right)$$

$$= 40 + 4(4\cdot 10)$$

$$= 40 + 4(40)$$

$$= 40 + 160$$

$$= \boxed{200}$$

50 $P(x) = F(x)G(x)$ & $Q(x) = \dfrac{F(x)}{G(x)}$

HOW TO: use graph to determine that:

$F(2) = 3$
$G(2) = 2$
$F'(2) = 0$
$G'(2) = 1/2$

and

$F(7) = 5$
$G(7) = 1$
$F'(7) = 1/4$
$G'(7) = -2/3$

a. Find $P'(2)$

$$P'(x) = G(x) \cdot F'(x) + F(x) \cdot G'(x)$$

$$P'(2) = G(2)F'(2) + F(2)G'(2)$$

$$= (2)(0) + (3)(1/2)$$

$$= 0 + 3/2 = \boxed{3/2}$$

---

b. Find $Q'(7)$

$$Q'(x) = \frac{G(x) \cdot F'(x) - F(x) \cdot G'(x)}{[G(x)]^2}$$

$$Q'(7) = \frac{G(7)F'(7) - F(7)G'(7)}{[G(7)]^2}$$

$$= \frac{(1)(1/4) - (5)(-2/3)}{(1)^2}$$

$$= \frac{1/4 + 10/3}{1} = \frac{3/12 + 40/12}{1} = \boxed{43/12}$$

(49) $u(x) = f(x)\,g(x)$ & $v(x) = \dfrac{f(x)}{g(x)}$

HOW TO: use graph to determine that:

$f(1) = 2$ | $f(5) = 3$
$g(1) = 1$ | and | $g(5) = 2$
$f'(1) = 2$ | $f'(5) = -1/3$
$g'(1) = -1$ | $g'(5) = 3/2$

a. Find $u'(1)$

$$u'(x) = g(x)\cdot f'(x) + f(x)\cdot g'(x)$$

$$u'(1) = g(1)\cdot f'(1) + f(1)\cdot g'(1)$$

$$= (1)(2) + (2)(-1)$$

$$= 2 - 2 = \boxed{0}$$

---

b. Find $v'(5)$

$$v'(x) = \frac{g(x)\,f'(x) - f(x)\,g'(x)}{[g(x)]^2}$$

$$v'(5) = \frac{g(5)\cdot f'(5) - f(5)\cdot g'(5)}{[g(5)]^2}$$

$$= \frac{(2)(-1/3) - (3)(3/2)}{(2)^2}$$

$$= \frac{-2/3 - 9/2}{4} = \frac{-4/6 - 27/6}{4} = \frac{-31/6}{4} = \boxed{\frac{-31}{24}}$$