

purple is
unsimplified answer

red is
simplified answer

blue is
original problem
rewritten

Calculus Worksheet
Chain Rule Practice #1

Find the derivative of each of the following functions.

1. $y = (x^2 + 4x + 6)^5$

$$y' = 5(x^2 + 4x + 6)^4 \cdot (2x + 4)$$

$$y' = 5(2x + 4)(x^2 + 4x + 6)^4$$

2. $y = \tan 3x$

$$\frac{dy}{dx} = \sec^2(3x) \cdot 3$$

$$\frac{dy}{dx} = 3\sec^2(3x)$$

3. $f(x) = (x^3 - 5x)^4$

$$f'(x) = 4(x^3 - 5x)^3 \cdot (3x^2 - 5)$$

$$f'(x) = 4(3x^2 - 5)(x^3 - 5x)^3$$

4. $y = 4 \sec 5x$

$$y' = 4 \tan(5x) \sec(5x) \cdot 5$$

$$y' = 20 \tan(5x) \sec(5x)$$

5. $f(x) = (3x - 2)^{10} (5x^2 - x + 1)^{12}$

$$f' = \underbrace{(5x^2 - x + 1)^{12}}_g \cdot \underbrace{10(3x - 2)^9 \cdot 3}_{f'}$$

$$+ \underbrace{(3x - 2)^{10}}_f \cdot \underbrace{12(5x^2 - x + 1)^{11} \cdot (10x - 1)}_{g'}$$

$$f' = 30(5x^2 - x + 1)^{12}(3x - 2)^9 + 12(3x - 2)^{10}(5x^2 - x + 1)^{11}(10x - 1)$$

6. $y = \cos(x^3)$

$$y' = -\sin(x^3) \cdot 3x^2$$

$$y' = -3x^2 \sin(x^3)$$

7. $f(x) = (6x^2 + 5)^3 (x^3 - 7)^4$

$$\frac{dy}{dx} = \underbrace{(x^3 - 7)^4}_f \cdot \underbrace{3(6x^2 + 5)^2 \cdot 12x}_{f'}$$

$$+ \underbrace{(6x^2 + 5)^3}_g \cdot \underbrace{4(x^3 - 7)^3 \cdot 3x^2}_{g'}$$

8. $y = \cos^3 x = (\cos x)^3$

$$\frac{dy}{dx} = 3 \cos^2 x \cdot -\sin x$$

$$\frac{dy}{dx} = -3 \sin x \cos^2 x$$

9. $y = (2x^2 - 6x + 1)^{-8}$

$$\frac{dy}{dx} = -8(2x^2 - 6x + 1)^{-9} \cdot (4x - 6)$$

$$\frac{dy}{dx} = -8(4x - 6)(2x^2 - 6x + 1)^{-9}$$

10. $f(x) = (1 + \cos^2 x)^6 = (1 + (\cos x)^2)^6$

$$\frac{dy}{dx} = 6(1 + \cos^2 x)^5 \cdot (2 \cos x \cdot -\sin x)$$

$$\frac{dy}{dx} = -6(2 \cos x \cdot \sin x)(1 + \cos^2 x)^5$$

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Calculus Worksheet
Chain Rule Practice #2

Find the derivative of each of the following functions.

1. $y = \sqrt{x^2 - 7x} = (x^2 - 7x)^{1/2}$

$$y' = \frac{1}{2}(x^2 - 7x)^{-1/2} \cdot (2x - 7)$$

$$y' = \frac{2x - 7}{2\sqrt{x^2 - 7x}}$$

3. $y = \frac{1}{(x^2 - 2x - 5)^4} = (x^2 - 2x - 5)^{-4}$

$$y' = -4(x^2 - 2x - 5)^{-5} \cdot (2x - 2)$$

$$y' = \frac{-4(2x - 2)}{(x^2 - 2x - 5)^5}$$
$$= \frac{-8x + 8}{(x^2 - 2x - 5)^5}$$

5. $f(x) = \left(x - \frac{1}{x}\right)^2 = (x - x^{-1})^{3/2}$

$$f' = \frac{3}{2}(x - x^{-1})^{1/2} \cdot (1 + x^{-2})$$

$$f' = \frac{3}{2}\sqrt{x - \frac{1}{x}} \cdot (1 + x^{-2})$$

7. $y = \left(\frac{x-6}{x+7}\right)^3$

$$\frac{dy}{dx} = 3\left(\frac{x-6}{x+7}\right)^2 \cdot \left(\frac{(x+7)(1) - (x-6)(1)}{(x+7)^2}\right)$$

$$\frac{dy}{dx} = 3\left(\frac{x-6}{x+7}\right)^2 \cdot \frac{13}{(x+7)^2} = \frac{39(x-6)^2}{(x+7)^4}$$

9. $y = \frac{1}{\sqrt[5]{2x-1}} = (2x-1)^{-1/5}$

$$\frac{dy}{dx} = -\frac{1}{5}(2x-1)^{-6/5} \cdot 2$$

$$\frac{dy}{dx} = -\frac{2}{5}(2x-1)^{-6/5}$$

11. $y = \sin^3(2x+3) = (\sin(2x+3))^3$

$$y' = 3(\sin(2x+3))^2 \cdot \cos(2x+3) \cdot 2$$

$$y' = 6 \sin^2(2x+3) \cos(2x+3)$$

2. $y = \tan(x^2) + \tan^2 x = \tan(x^2) + (\tan x)^2$

$$y' = \sec^2(x^2) \cdot 2x + 2 \tan x \cdot \sec^2 x$$

$$y' = 2x \sec^2(x^2) + 2 \tan x \sec^2 x$$

4. $y = \cos(\tan x)$

$$\frac{dy}{dx} = -\sin(\tan x) \cdot \sec^2 x$$

$$\frac{dy}{dx} = -\sin(\tan x) \sec^2 x$$

6. $y = \sin^3 x + \cos^3 x = (\sin x)^3 + (\cos x)^3$

$$y' = 3 \sin^2 x \cdot \cos x + 3 \cos^2 x \cdot -\sin x$$

$$y' = 3 \sin^2 x \cos x - 3 \cos^2 x \sin x$$

8. $y = \sin^2(\cos(4x)) = (\sin(\cos(4x)))^2$

$$y' = 2(\sin(\cos(4x))) \cdot \cos(\cos(4x)) \cdot -\sin(4x) \cdot 4$$

$$y' = -8 \sin(\cos(4x)) \cos(\cos(4x)) \sin(4x)$$

10. $y = \frac{\sin^2 x}{\cos x} = \sin x \tan x$

$$y' = \tan x \cos x + \sin x \sec^2 x$$

$$y' = \tan x \cos x + \sin x \sec^2 x$$