1. $y = x^2 + xy$

$$\frac{dy}{dx} = 2x + y \cdot 1 + x \cdot \frac{dy}{dx}$$

$$\frac{dy}{dx} - x\frac{dy}{dx} = 2x + y$$

$$\frac{dy}{dx}(1 - x) = 2x + y$$

$$\frac{dy}{dx} = \frac{2x+y}{1-x}$$

2. $x^2y + y = 3$

$$y \cdot 2x + x^2\frac{dy}{dx} + \frac{dy}{dx} = 0$$

$$x^2\frac{dy}{dx} + \frac{dy}{dx} = -2xy$$

$$\frac{dy}{dx}(x^2+1) = -2xy$$

$$\frac{dy}{dx} = \frac{-2xy}{x^2+1}$$

3. $x^{1/4} + y^{1/4} = 2$

$$\frac{1}{4}x^{-3/4} + \frac{1}{4}y^{-3/4}\frac{dy}{dx} = 0$$

$$\frac{1}{4}y^{-3/4}\frac{dy}{dx} = -\frac{1}{4}x^{-3/4}$$

$$\frac{dy}{dx} = \frac{-\frac{1}{4}x^{-3/4}}{\frac{1}{4}y^{-3/4}}$$

$$\frac{dy}{dx} = \frac{-y^{3/4}}{x^{3/4}}$$

4. $x^{1/3} + y^{1/3} = 7$

$$\frac{1}{3}x^{-2/3} + \frac{1}{3}y^{-2/3}\frac{dy}{dx} = 0$$

$$\frac{1}{3}y^{-2/3}\frac{dy}{dx} = -\frac{1}{3}x^{-2/3}$$

$$\frac{dy}{dx} = \frac{-\frac{1}{3}x^{-2/3}}{\frac{1}{3}y^{-2/3}}$$

$$\frac{dy}{dx} = \frac{-y^{2/3}}{x^{2/3}}$$

5. $\sqrt{x} + \sqrt{y} = 25$

$$\frac{1}{2}x^{-1/2} + \frac{1}{2}y^{-1/2}\cdot\frac{dy}{dx} = 0$$

$$\frac{1}{2}y^{-1/2}\frac{dy}{dx} = -\frac{1}{2}x^{-1/2}$$

$$\frac{dy}{dx} = \frac{-\frac{1}{2}x^{-1/2}}{\frac{1}{2}y^{-1/2}}$$

$$\frac{dy}{dx} = \frac{-y^{1/2}}{x^{1/2}} = -\sqrt{\frac{y}{x}}$$

6. $x^2 + y^2 = 1.1$

$$2x + 2y\frac{dy}{dx} = 0$$

$$2y\frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = \frac{-2x}{2y}$$

$$\frac{dy}{dx} = -x/y$$

7. $x^3 + y^3 = \sqrt{5}$

$3x^2 + 3y^2 \frac{dy}{dx} = 0$

$3y^2 \cdot \frac{dy}{dx} = -3x^2$

$\frac{dy}{dx} = \frac{-3x^2}{3y^2}$

$\frac{dy}{dx} = -x^2/y^2$

8. $x + \sin y = y + 1$

$1 + \cos y \cdot \frac{dy}{dx} = \frac{dy}{dx}$

$\cos y \cdot \frac{dy}{dx} - \frac{dy}{dx} = -1$

$\frac{dy}{dx}(\cos y - 1) = -1$

$\frac{dy}{dx} = \frac{-1}{\cos y - 1}$

17. $1 - xy = x - y^2$

$0 - x\frac{dy}{dx} + y \cdot -1 = 1 - 2y\frac{dy}{dx}$

$-x\frac{dy}{dx} + 2y\frac{dy}{dx} = y + 1$

$\frac{dy}{dx}(-x + 2y) = y + 1$

$\frac{dy}{dx} = \frac{y+1}{-x+2y}$

$\frac{d^2y}{dx^2} = \frac{(-x+2y)(\frac{dy}{dx}) - (y+1)(-1+2\frac{dy}{dx})}{(-x+2y)^2}$ (sub $\frac{dy}{dx}$, sub $\frac{dy}{dx}$)

$\frac{d^2y}{dx^2} = \frac{(-x+2y)(\frac{y+1}{-x+2y}) - (y+1)(-1+2(\frac{y+1}{-x+2y}))}{(-x+2y)^2}$

18. $x - y = (x+y)^2$

$1 - \frac{dy}{dx} = 2(x+y)^1 \cdot (1 + \frac{dy}{dx})$

$1 - \frac{dy}{dx} = (2x + 2y)(1 + \frac{dy}{dx})$

$1 - \frac{dy}{dx} = 2x + 2x\frac{dy}{dx} + 2y + 2y\frac{dy}{dx}$

$-\frac{dy}{dx} - 2x\frac{dy}{dx} - 2y\frac{dy}{dx} = -1 + 2x + 2y$

$\frac{dy}{dx}(-1 - 2x - 2y) = -1 + 2x + 2y$

$\frac{dy}{dx} = \frac{-1+2x+2y}{-1-2x-2y}$

$\frac{d^2y}{dx^2} = \frac{(-1-2x-2y)(2 + 2\frac{dy}{dx}) - (-1+2x+2y)(-2-2\frac{dy}{dx})}{(-1-2x-2y)^2}$ (sub $\frac{dy}{dx}$, sub $\frac{dy}{dx}$)

$\frac{d^2y}{dx^2} = \frac{(-1-2x-2y)(2 + 2(\frac{-1+2x+2y}{-1-2x-2y})) - (-1+2x+2y)(-2-2(\frac{-1+2x+2y}{-1-2x-2y}))}{(-1-2x-2y)^2}$

21. $x^2 + y^2 - xy + 3x - 9 = 0$

$2x + 2y\frac{dy}{dx} - x\frac{dy}{dx} + y(-1) + 3 = 0$

$2y\frac{dy}{dx} - x\frac{dy}{dx} = -2x + y - 3$

$\frac{dy}{dx}(2y - x) = -2x + y - 3$

$\frac{dy}{dx} = \frac{-2x + y - 3}{2y - x}$

---

22. $\frac{dy}{dx} = -\frac{-2x + y - 3}{2y - x} = 0$, $-2x + y - 3 = 0$

(0, 3) and (−4, −5)

23. (2, 1) and (−6, −3)

24. $\frac{d^2y}{dx^2} = \frac{(2y - x)(-2 + \frac{dy}{dx}) - (-2x + y - 3)(2\frac{dy}{dx} - 1)}{(2y - x)^2}$ (sub)

$\frac{d^2y}{dx^2} = \frac{(2y - x)(-2 + (\frac{-2x + y - 3}{2y - x})) - (-2x + y - 3)(2(\frac{-2x + y - 3}{2y - x}) - 1)}{(2y - x)^2}$

Kuta Software - Infinite Calculus

Name Key

Date _____ Period ____

Implicit Differentiation

For each problem, use implicit differentiation to find $\frac{dy}{dx}$ in terms of x and y.

1) $2x^3 = 2y^2 + 5$

$6x^2 = 4y\frac{dy}{dx}$

$\boxed{\frac{dy}{dx} = \frac{6x^2}{4y} = \frac{3x^2}{2y}}$

2) $3x^2 + 3y^2 = 2$

$6x + 6y\frac{dy}{dx} = 0$

$6y\frac{dy}{dx} = -6x$

$\boxed{\frac{dy}{dx} = \frac{-6x}{6y} = -\frac{x}{y}}$

3) $5y^2 = 2x^3 - 5y$

$10y\frac{dy}{dx} = 6x^2 - 5\frac{dy}{dx}$

$10y\frac{dy}{dx} + 5\frac{dy}{dx} = 6x^2$

$\boxed{\frac{dy}{dx} = \frac{6x^2}{10y+5}}$

4) $4x^2 = 2y^3 + 4y$

$8x = 6y^2\frac{2dy}{dx} + 4\frac{dy}{dx}$

$8x = \frac{dy}{dx}(6y^2 + 4)$

$\boxed{\frac{dy}{dx} = \frac{8x}{6y^2+4}}$

5) $5x^3 = -3xy + 2$

$15x^2 = (-3x)(\frac{dy}{dx}) + (y)(-3)$

$3x\frac{dy}{dx} = -3y - 15x^2$

$\boxed{\frac{dy}{dx} = \frac{-3y-15x^2}{3x} = \frac{-y-5x^2}{x}}$

6) $1 = 3x + 2x^2y^2$

$0 = 3 + (2x^2)(2y\frac{dy}{dx}) + (y^2)(4x)$

$-2x^2 2y\frac{dy}{dx} = 3 + y^2 4x$

$\boxed{\frac{dy}{dx} = \frac{3+y^2 4x}{-2x^2 2y} = \frac{3+4xy^2}{-4x^2y}}$

11) $\sin(2x^2y^3) = 3x^3 + 1$

$\cos(2x^2y^3)\cdot(y^3\cdot 4x + 2x^2\cdot 3y^2\frac{dy}{dx}) = 9x^2$

$4xy^3\cos(2x^2y^3) + 6x^2y^2\cos(2x^2y^3)\frac{dy}{dx} = 9x^2$

$6x^2y^2\cos(2x^2y^3)\frac{dy}{dx} = 9x^2 - 4xy^3\cos(2x^2y^3)$

$\boxed{\frac{dy}{dx} = \frac{9x^2 - 4xy^3\cos(2x^2y^3)}{6x^2y^2\cos(2x^2y^3)}}$

12) $3x^2 + 3 = \ln 5xy^2$

rewrite: $3x^2 + 3 = \ln 5 + \ln x + \ln y^2$

differentiate: $6x = 0 + \frac{1}{x} + \frac{1}{y^2}\cdot 2y\frac{dy}{dx}$

$-\frac{2y}{y^2}\frac{dy}{dx} = -6x + \frac{1}{x}$

$-\frac{2}{y}\frac{dy}{dx} = -6x + \frac{1}{x}$

$\boxed{\frac{dy}{dx} = (-6x + \frac{1}{x})\cdot\frac{-y}{2}}$

For each problem, use implicit differentiation to find $\frac{d^2y}{dx^2}$ in terms of x and y.

13) $4y^2 + 2 = 3x^2$

$8y\frac{dy}{dx} = 6x$

$\boxed{\frac{dy}{dx} = \frac{6x}{8y}}$

$\frac{d^2y}{dx^2} = \frac{(8y)(6) - (6x)(8\frac{dy}{dx})}{(8y)^2}$

$\boxed{\frac{d^2y}{dx^2} = \frac{48y - 48x(\frac{6x}{8y})}{(8y)^2}}$

14) $5 = 4x^2 + 5y^2$

$0 = 8x + 10y\frac{dy}{dx}$

$-10y\frac{dy}{dx} = 8x$

$\boxed{\frac{dy}{dx} = -\frac{8x}{10y} = \frac{-4x}{5y}}$

$\frac{d^2y}{dx^2} = \frac{(5y)(4) - (4x)(5\frac{dy}{dx})}{(5y)^2}$

$= \frac{20y + 20x(\frac{4x}{5y})}{(5y)^2}$