U3H6

Find the derivative of y with respect to the appropriate variable.

1. $y = \cos^{-1}(x^2)$

2. $y = \arcsin\sqrt{2t}$

3. $y = \sin^{-1}\frac{3}{t^2}$

4. $y = x\sin^{-1}x + \sqrt{1-x^2}$

5. (a) Find an equation for the tangent to the graph of $y = \tan x$ at the point $(\pi/4, 1)$.

(b) Find an equation for the tangent to the graph of $y = \tan^{-1}x$ at the point $(1, \pi/4)$.

6. Let $f(x) = \cos x + 3x$.

(a) Find $f(0)$

b. Find $f'(0)$

7. Which of the following is $\frac{d}{dx}\sin^{-1}\left(\frac{x}{2}\right)$?

(a) $\frac{1}{\sqrt{4-x^2}}$ **(b)** $-\frac{1}{\sqrt{4-x^2}}$ **(c)** $\frac{2}{\sqrt{4-x^2}}$

8. Which of the following is $\frac{d}{dx}\arctan(3x)$?

(a) $\frac{1}{1+9x^2}$ **(b)** $-\frac{3}{1+9x^2}$ **(c)** $\frac{3}{1+9x^2}$

Which of the following is the slope of the tangent line to $y = \tan^{-1}(2x)$ at $x = 1$?

(a) $\frac{2}{5}$ **(b)** $\frac{1}{5}$ **(c)** 5 **(d)** $\frac{5}{2}$ **(e)** $-\frac{2}{5}$

Find the derivative of y with respect to the appropriate variable.

1. $y = \cos^{-1}(x^2)$

$y' = \frac{-1}{\sqrt{1-(x^2)^2}} \cdot 2x$

2. $y = \arcsin\sqrt{2t}$

$y' = \frac{1}{\sqrt{1-(\sqrt{2}t)^2}} \cdot \sqrt{2}$

3. $y = \sin^{-1}\frac{3}{t^2}$

$y' = \frac{1}{\sqrt{1-(3/t^2)^2}} \cdot -6t^{-3}$

4. $y = x\sin^{-1}x + \sqrt{1-x^2}$

$y' = (\sin^{-1}x)(1) + (x)\left(\frac{1}{\sqrt{1-x^2}}\right) + \frac{1}{2}(1-x^2)^{-1/2}(-2x)$

$= \sin^{-1}x + \frac{x}{\sqrt{1-x^2}} + \frac{-2x}{2\sqrt{1-x^2}}$

$= \sin^{-1}x$

5. (a) Find an equation for the tangent to the graph of $y = \tan x$ at the point $(\pi/4, 1)$. point

slope: $y' = \sec^2 x$

$y'(\pi/4) = \sec^2(\pi/4)$

$y - 1 = \sec^2(\pi/4)(x - \pi/4)$

(b) Find an equation for the tangent to the graph of $y = \tan^{-1}x$ at the point $(1, \pi/4)$. point

slope: $y' = \frac{1}{1+x^2}$

$y'(1) = \frac{1}{1+(1)^2} = \frac{1}{2}$

$y - \pi/4 = \frac{1}{2}(x-1)$

6. Let $f(x) = \cos x + 3x$.

(a) Find $f(0)$

$f(0) = \cos 0 + 3(0)$

$= 1 + 0$

$= 1$

b. Find $f'(0)$

$f'(x) = -\sin x + 3$

$f'(0) = -\sin(0) + 3$

$= 0 + 3 = 3$

7. Which of the following is $\frac{d}{dx}\sin^{-1}\left(\frac{x}{2}\right)$?

(a) $\frac{1}{\sqrt{4-x^2}}$ (b) $-\frac{1}{\sqrt{4-x^2}}$ (c) $\frac{2}{\sqrt{4-x^2}}$

$y' = \frac{1}{\sqrt{1-(\frac{x}{2})^2}} \cdot \frac{1}{2} = \frac{1}{2\sqrt{1-\frac{x^2}{4}}}$

$= \frac{1}{2\sqrt{\frac{4}{4}-\frac{x^2}{4}}} = \frac{1}{2\sqrt{\frac{1}{4}(4-x^2)}}$

$= \frac{1}{2\cdot\frac{1}{2}\sqrt{4-x^2}} = \frac{1}{\sqrt{4-x^2}}$

8. Which of the following is $\frac{d}{dx}\arctan(3x)$?

(a) $\frac{1}{1+9x^2}$ (b) $-\frac{3}{1+9x^2}$ (c) $\frac{3}{1+9x^2}$

$y' = \frac{1}{1+(3x)^2} \cdot 3 = \frac{1}{1+9x^2} \cdot 3$

$= \frac{3}{1+9x^2}$

Which of the following is the slope of the tangent line to $y = \tan^{-1}(2x)$ at $x = 1$?

(a) $2/5$ (b) $1/5$ (c) 5 (d) $5/2$ (e) $-2/5$

$y' = \frac{1}{1+(2x)^2} \cdot 2$

$y'(1) = \frac{2}{1+4} = \frac{2}{5}$