Name *key* Date ________ Period ______

Worksheet 2.6—The Chain Rule

Short Answer

Show all work, including rewriting the original problem in a more useful way. No calculator unless otherwise stated.

1. Find the derivative of the following functions with respect to the independent variable. (You do not need to simplify your final answers here.)

(a) $y=(2x-7)^3$

$\frac{dy}{dx}=3(2x-7)^2\cdot 2$

(b) $y=\dfrac{1}{t^2+3t-1}=(t^2+3t-1)^{-1}$

$y'=-1(t^2+3t-1)^{-2}\cdot(2t+3)$

(c) $y=\left(\dfrac{1}{t-3}\right)^2$

$y'=2\left(\frac{1}{t-3}\right)^1\cdot\left(\frac{(t-3)(0)-(1)(1)}{(t-3)^2}\right)$

or rewrite: $y=(t-3)^{-2}$

$y'=-2(t-3)^{-3}\cdot 1$

(d) $y=\csc^3\left(\dfrac{3x}{2}\right)=\left(\csc\left(\frac{3x}{2}\right)\right)^3$

$y'=3\left(\csc\left(\frac{3x}{2}\right)\right)^2\cdot -\csc\left(\frac{3x}{2}\right)\cot\left(\frac{3x}{2}\right)\cdot\frac{3}{2}$

(e) $y=3\sec^2(\pi t-1)=3(\sec(\pi t-1))^2$

$\frac{dy}{dt}=6(\sec(\pi t-1))^1\cdot\sec(\pi t-1)\tan(\pi t-1)\cdot\pi$

(f) $y=\sin\sqrt[3]{x}+\sqrt[3]{\sin x}=\sin(x^{1/3})+(\sin x)^{1/3}$

$y'=\cos(x^{1/3})\cdot\frac{1}{3}x^{-2/3}+\frac{1}{3}(\sin x)^{-2/3}\cdot\cos x$

(g) $y=\overbrace{x^2}^{f}\overbrace{\tan\frac{1}{x}}^{g}=x^2\tan(x^{-1})$

$y'=(\tan(x^{-1}))(2x)+(x^2)(\sec^2(x^{-1})\cdot -x^{-2})$

(h) $r=\overbrace{\sec 2\theta}^{f}\,\overbrace{\tan 2\theta}^{g}$

$\frac{dr}{d\theta}=(\tan 2\theta)(\sec 2\theta\tan 2\theta\cdot 2)+(\sec 2\theta)(\sec^2 2\theta\cdot 2)$

(i) $f(x)=\sqrt[3]{\csc^5 7}=(\csc 7)^{5/3}$

$f'=0$

2. Find the equation of the tangent line for each of the following at the indicated point.

(a) $s(t)=\sqrt{t^2+2t+8}$ at $x=2$

pt: $\boxed{(2,4)}$

slope: $s'(t)=\frac{1}{2}(t^2+2t+8)^{-1/2}\cdot(2t+2)$

$s'(2)=\frac{1}{2}(2^2+2(2)+8)^{-1/2}\cdot(2\cdot 2+2)$

$=\frac{1}{2}(16)^{-1/2}\cdot 6$

$=\frac{1}{2\sqrt{16}}\cdot 6=\frac{1}{8}\cdot 6=\boxed{\frac{3}{4}}$

$\boxed{y-4=\frac{3}{4}(x-2)}$

(b) $f(t)=\dfrac{3t+2}{t-1}$ at $(0,-2)$

pt: $\boxed{(0,-2)}$

slope: $f'(t)=\dfrac{(t-1)(3)-(3t+2)(1)}{(t-1)^2}$

$f'(0)=\dfrac{(0-1)(3)-(3\cdot 0+2)(1)}{(0-1)^2}$

$=\dfrac{-3-2}{1}=\boxed{-5}$

$\boxed{y+2=-5(x-0)}$

3. Determine the point(s) in the interval $(0,2\pi)$ at which the graph of $f(x)=2\cos x+\sin 2x$ has a $\boxed{\text{horizontal tangent.}}$ means slope $=0\ldots$ $\frac{dy}{dx}=0$

$y=2\cos x+\sin 2x$

$\frac{dy}{dx}=-2\sin x+2\cos 2x$

$0=-2\sin x+2\cos 2x$

$0=\sin x-\underline{\cos 2x}$ *double angle formula

$0=\sin x-(\underline{1-2\sin^2 x})$

$0=\sin x-1+2\sin^2 x$

$0=2\sin^2 x+\sin x-1$

Factor: $0=(2\sin x-1)(\sin x+1)$

$2\sin x-1=0$

$\sin x=1/2$

$\boxed{x=\pi/6,\ 5\pi/6}$

$\sin x+1=0$

$\sin x=-1$

$\boxed{x=3\pi/2}$

4. Find the second derivative of each of the following functions. Remember to simplify early and often.

(a) $f(x)=2(x^2-1)^3$

$f'(x)=6(x^2-1)^2\cdot(2x)$

$=\underbrace{12x}_{f}\underbrace{(x^2-1)^2}_{g}$

$f''(x)=(x^2-1)^2(12)+(12x)(2(x^2-1)\cdot 2x)$

$=12(x^2-1)^2+\quad x^2(x^2-1)$

(b) $f(x)=\sin(x^2)$

$f'(x)=\cos(x^2)\cdot 2x$

$=\underbrace{2x}_{f}\underbrace{\cos x^2}_{g}$

$f''(x)=(\cos x^2)(2)+(2x)(-\sin x^2)(2x)$

$=2\cos x^2-4x^2\sin x^2$

5. If $h(x)=\tan(2x)$, evaluate $h''(x)$ at $\left(\frac{\pi}{6},\sqrt{3}\right)$. Simplify early and often.

$h'(x) = \sec^2(2x)\cdot 2$

$= 2\sec^2(2x)$

$= 2(\sec 2x)^2$

$h''(x) = 4(\sec 2x)'\cdot \sec 2x \tan 2x \cdot 2$

$h''(\pi/6) = 4(\sec(2\cdot\pi/6)\cdot \sec 2\cdot\pi/6 \tan 2\cdot\pi/6\cdot 2$

$= 4\left(\frac{1}{\cos\pi/3}\right)\left(\frac{1}{\cos\pi/3}\right)\frac{\sin\pi/3}{\cos\pi/3}\cdot 2$

$h''(\pi/6) = 4\left(\frac{1}{1/2}\right)\left(\frac{1}{1/2}\right)\frac{\sqrt{3}/2}{1/2}\cdot 2$

$= 4(2)(2)(\sqrt{3})\cdot(2)$

$= \boxed{32\sqrt{3}}$

6. If $g(5)=-3$, $g'(5)=6$, $h(5)=3$, and $h'(5)=-2$, find $f'(5)$ (if possible) for each of the following. If it is not possible, state what additional information is required.

(a) $f(x)=\dfrac{g(x)}{h(x)}$

$f'(x) = \dfrac{h(x)g'(x) - g(x)h'(x)}{[h(x)]^2}$

$f'(5) = \dfrac{(3)(6) - (-3)(-2)}{(3)^2}$

$= \dfrac{18-6}{6}$

$= \boxed{2}$

(b) $f(x)=g(h(x))$

$f'(x) = g'(h(x))\cdot h'(x)$

$f'(5) = g'(h(5))\cdot h'(5)$

$= g'(3)\cdot -2$

$= \boxed{?}\cdot -2$

= Not enough info... missing value of $g'(3)$.

(c) $f(x)=g(x)h(x)$

$f'(x) =$

$h(x)g'(x) + g(x)h'(x)$

$f'(5) = (3)(6) + (-3)(-2)$

$= 18+6$

$= \boxed{24}$

(d) $f(x)=[g(x)]^3$

$f'(x) = 3(g(x))^2\cdot g'(x)$

$f'(5) = 3(g(5))^2\cdot g'(5)$

$= 3(-3)^2\cdot 6$

$= 3(9)\cdot 6$

$= 27\cdot 6$

$= \boxed{163}$

(e) $f(x)=g(x+h(x))$

$f'(x) = g'(x+h(x))\cdot(1+h'(x))$

$f'(5) = g'(5+h(5))\cdot(1+h'(5))$

$= g'(5+3)\cdot(1-2)$

$= g'(8)\cdot(-1)$

$= \boxed{?}\cdot -1$

= Not enough info... missing value of $g'(8)$.

(f) $f(x)=(g(x)+h(x))^{-2}$

$f'(x) = -2(g(x)+h(x))^{-3}\cdot(g'(x)+h'(x))$

$f'(5) = -2(g(5)+h(5))^{-3}\cdot(g'(5)+h'(5))$

$= -2(-3+3)^{-3}\cdot(6-2)$

$= \dfrac{-2}{(-3+3)^3}\cdot 4$

$= \dfrac{-2}{(0)^3}\cdot 4$

$= \dfrac{-2}{0}\cdot 4$

$= \boxed{\text{DNE}}$

Multiple Choice

rewrite: $(x^2+3)^{-1/2}$

A 14. If $f(x)=\dfrac{1}{\sqrt{x^2+3}}$, find $f'(x)$.

(A) $f'(x)=-\dfrac{x}{\sqrt{(x^2+3)^3}}$

(B) $f'(x)=\dfrac{x}{\sqrt{x^2+3}}$

(C) $f'(x)=-\dfrac{x}{(x^2+3)\sqrt{2x}}$

(D) $f'(x)=-\dfrac{1}{2\sqrt{(x^2+3)^3}}$

(E) $f'(x)=-\dfrac{x^2+3x}{x^2+3}$

$f'(x) = -\frac{1}{2}(x^2+3)^{-3/2} \cdot 2x$

$= \dfrac{-\cancel{2}x}{\cancel{2}(x^2+3)^{3/2}}$

$= \dfrac{-x}{\sqrt{(x^2+3)^3}}$

C 15. If $g(x)=\overbrace{(1-x)^3}^{f}\overbrace{(4x+1)}^{g}$, then $g'(x)=$

(A) $-12(1-x)^2$

(B) $(1-x)^2(1+8x)$

(C) $(1-x)^2(1-16x)$

(D) $3(1-x)^2(4x+1)$

(E) $(1-x)^2(16x+7)$

$g'(x) = (4x+1)(3(1-x)^2\cdot -1) + (1-x)^3(4)$

$= -3(4x+1)(1-x)^2 + 4(1-x)^3$

$= (1-x)^2[-3(4x+1) + 4(1-x)]$

$= (1-x^2)[-12x-3+4-4x]$

$= (1-x^2)(-16x+1)$

D 16. $\dfrac{d}{dx}\left[\left(\dfrac{x^2-3}{5x^2-9}\right)^5\right] = 5\left(\dfrac{x^2-3}{5x^2-9}\right)^4 \cdot \left(\dfrac{(5x^2-9)(2x)-(x^2-3)(10x)}{(5x^2-9)^2}\right)$

(A) $\dfrac{10x(x^2-3)^4(10x^2-17)}{(5x^2-9)^6}$

(B) $\dfrac{-10x(x^2-3)^4(5x^2-16)}{(5x^2-9)^5}$

(C) $\dfrac{-240x(x^2-3)^4}{(5x^2-9)^6}$

(D) $\dfrac{60x(x^2-3)^4}{(5x^2-9)^6}$

(E) $\dfrac{100x(x^2-3)^4}{(5x^2-9)^6}$

$= \dfrac{5(x^2-3)^4}{(5x^2-9)^4} \cdot \left(\dfrac{\cancel{10x^3}-18x-\cancel{10x^3}+30x}{(5x^2-9)^2}\right)$

$= \dfrac{5(x^2-3)^4(12x)}{(5x^2-9)^6}$

$= \dfrac{60x(x^2-3)^4}{(5x^2-9)^6}$

D 17. A derivative of a function $f(x)$ is obtained using the chain rule. The result is $f'(x)=3\sec^3 x\tan x$. Which of the following could be $f(x)$?

I. $f(x)=-\pi+\frac{3}{4}\sec^4 x$ $\quad f'(x) = 0 + \frac{3}{4}\cdot 4(\sec x)^3 \cdot \sec x\tan x = 3\sec^4 x\tan x$ ✗

II. $f(x)=8+\sec^3 x$ $\quad f'(x) = 0 + 3(\sec x)^2 \cdot \sec x\tan x = 3\sec^3 x\tan x$ ✓

III. $f(x)=\sec x+\sec x\tan^2 x$ $\quad f'(x) = \sec x\tan x + (\tan^2 x)(\sec x\tan x) + (\sec x)\cdot 2\tan x\cdot\sec^2 x$

(A) I only (B) II only (C) III only (D) II and III only (E) I, II, and III

$= \sec x\tan x(1+\tan^2 x + 2\sec^2 x)$

$= \sec x\tan x(\sec^2 x + 2\sec^2 x)$

$= \sec x\tan x(3\sec^2 x)$

$= 3\sec^3 x\tan x$ ✓