① What is $\lim_{x\to\infty} \frac{x^2 - \cancel{6}}{\cancel{2} + \cancel{x} - 3x^2}$? $\rightarrow \frac{x^2}{-3x^2} = -\frac{1}{3}$

(A) -3 (B) $-\frac{1}{3}$ (C) $\frac{1}{3}$ (D) 2 (E) The limit does not exist.

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② The average rate of change of the function $f(x) = \left|x^2 - 2|x+2|\right|$ over the interval $-3 < x < -1$ is $\frac{f(-1) - f(-3)}{-1 - (-3)} = \frac{-6}{2}$

(A) -3 (B) -2 (C) -1 (D) 1 (E) 2

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③ Which of the following is true about the function f if $f(x) = \frac{(x-1)^2}{2x^2 - 5x + 3}$? $= \frac{(x-1)^2}{(2x-3)(x-1)}$

I. f is continuous at $x = 1$. ✗ no - discont. at $x=1$

II. The graph of f has a vertical asymptote at $x = 1$. ✗ no! ~~[illegible]~~ has a hole at $x=-1$

✓ III. The graph of f has a horizontal asymptote at $y = \frac{1}{2}$. yes! check $\lim_{x\to\pm\infty} f(x) = \frac{1}{2}$

(A) I only (B) II only (C) III only (D) II and III only (E) I, II, III

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④ An equation of the normal to the graph of $f(x) = \frac{x}{2x-3}$ at $(1, f(1))$ is

(A) $3x + y = 4$

(B) $3x + y = 2$

(C) $x - 3y = -2$

(D) $x - 3y = 4$

(E) $x + 3y = 2$

pt : $\boxed{(1,-1)}$

slope: $y' = \frac{(2x-3)(1) - (x)(2)}{(2x-3)^2}$

$y'(1) = \boxed{-3}$ ← tangent slope

$y + 1 = \frac{1}{3}(x - 1)$ ← now simplify to match an answer choice

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⑤ Consider the function $f(x) = \frac{6x}{a + x^3}$ for which $f'(0) = 3$. The value of a is

(A) 5

(B) 4

(C) 3

(D) 2

(E) 1

$f'(x) = \frac{(a+x^3)(6) - (6x)(3x^2)}{(a+x^3)^2}$

$f'(0) = \frac{(a+0^3)(6) - (6\cdot 0)(3\cdot 0^2)}{(a+0^3)^2} = 3$

$\frac{6a}{a^2} = 3$

so $a = 2$

(6)

If $h(x) = (x^2 - 4)^{3/4} + 1$, then the value of $h'(2)$ is

(A) 3

(B) 2

(C) 1

(D) 0

(E) does not exist

$h'(x) = \frac{3}{4}(x^2-4)^{-1/4} \cdot 2x$

$h'(2) = \frac{3}{4} \cdot \frac{1}{(2^2-4)^{1/4}} \cdot 4$

↑

bad!
0 in denom.

(7)

he fourth derivative of $f(x) = (2x - 3)^4$ is

A) $24(2^4)$

B) $24(2^3)$

C) $24(2x - 3)$

D) $24(2^5)$

E) 0

$f' = 4(2x-3)^3 \cdot 2 = 8(2x-3)^3$

$f'' = 24(2x-3)^2 \cdot 2 = 48(2x-3)^2$

$f''' = 96(2x-3)^1 \cdot 2 = 192(2x-3)$

$f'''' = 192 \cdot 2 = 384 = 24 \cdot 2^4$

2
16
24
64
32

(8)

f $g(x) = \frac{x-2}{x+2}$, then $g'(2) =$

A) 1

B) -1

C) $\frac{1}{4}$

D) $-\frac{1}{4}$

E) 0

$g' = \frac{(x+2)(1) - (x-2)(1)}{(x+2)^2}$

$g'(2) = \frac{4-0}{16} = \frac{1}{4}$

(9)

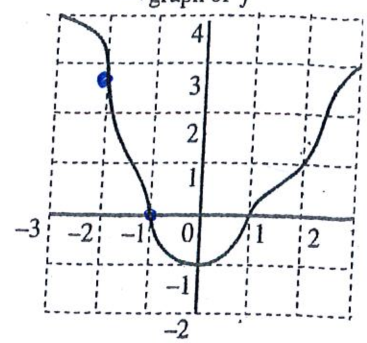
The graph of f', the derivative of a function f, is shown at the right.

Which of the following statements are true about the function f?

I. f is increasing on the interval $(-2, -1)$. TRUE

since $f' > 0$,
f is positive

graph of f'



(10)

The derivative of $\sqrt{x} - \frac{1}{x\sqrt[3]{x}}$ is

(A) $\frac{1}{2}x^{-1/2} - x^{-4/3}$

(B) $\frac{1}{2}x^{-1/2} + \frac{4}{3}x^{-7/3}$

(C) $\frac{1}{2}x^{-1/2} - \frac{4}{3}x^{-1/3}$

(D) $-\frac{1}{2}x^{-1/2} + \frac{4}{3}x^{-7/3}$

(E) $-\frac{1}{2}x^{-1/2} - \frac{4}{3}x^{-1/3}$

$y = x^{1/2} - x^{-4/3}$

$y' = \frac{1}{2}x^{-1/2} + \frac{4}{3}x^{-7/3}$

(11)

The function f is continuous at $x = 1$.

If $f(x) = \begin{cases} \frac{\sqrt{x+3} - \sqrt{3x+1}}{x-1} & \text{for } x \neq 1 \\ k & \text{for } x = 1 \end{cases}$ then $k =$

(A) 0 (B) 1 (C) $\frac{1}{2}$ (D) $-\frac{1}{2}$

$\frac{\sqrt{x+3} - \sqrt{3x+1}}{x-1} = K$

to solve, multiply by conjugate
and plug in $x = 1$