**AP Calculus**

## Worksheet: Tangents, Normals, and Tangent Line Approximations

There are several forms of linear equations but one of the more useful forms in Calculus is the point-slope form:

$y - y_1 = m(x - x_1)$ where m = the slope of the line and the point $(x_1,\ y_1)$ is on the line.

Since we know that the slope of the tangent line to a curve at the point $(a,\ f(a))$ is f ' (a), we can rewrite the equation above as

$$y - f(a) = f'(a)(x - a)$$

The normal (perpendicular) line to a curve has a slope that is the negative reciprocal of the tangent line slope. So we can write the equation of the normal line as

$$y - f(a) = \frac{-1}{f'(a)}(x - a)$$

In calculus, we are often interested in approximating a function using the tangent line. Because of the concept of local linearization, the tangent line function and the function itself have values that are similar for points very near the point of tangency. For the purpose of approximating function values we can rewrite the tangent line equation as

$$y \approx f(a) + f'(a)(x - a)$$

**Exercises:** All answers for #1 are is Standard Form

1. An equation of the line tangent to the graph of $y = \frac{2x+3}{3x-2}$ at the point (1, 5) is

(A) 13x - y = 8 (B) 13x + y = 18 (C) x - 13y = 64

(D) x + 13y = 66 (E) -2x + 3y = 13

Pt: $\boxed{(1,5)}$

Slope: $y' = \frac{(3x-2)(2) - (2x+3)(3)}{(3x-2)^2}$

$y'(1) = \boxed{-13}$

$y - 5 = -13(x-1)$
$y - 5 = -13x + 13$
$y = -13x + 18$
$13x + y = 18$
(standard form)



All answers for #2 are in slope-intercept form

2. An equation of the line tangent to the graph of y = x + cos x at the point (0, 1) is

(A) y = 2x + 1 (B) y = x + 1 (C) y = x (D) y = x - 1 (E) y = 0

Pt: $\boxed{(0,1)}$

Slope: $y' = 1 - \sin x$

$y'(0) = \boxed{1}$

$y - 1 = 1(x - 0)$
$y - 1 = x$
$y = x + 1$
(slope-intercept form)

*need calculator*

3. Which of the following is an equation of the line tangent to the graph of $f(x) = x^4 + 2x^2$ at the point where $\boxed{f'(x) = 1?}$ ← slope

(A) y = 8x - 5 (B) y = x + 7 (C) y = x + 0.763

(D) y = x - 0.122 (E) y = x - 2.146

$f'(x) = 4x^3 + 4x = 1$

$x = 0.237$

Pt: $\boxed{(0.237, 0.115)}$

Slope: $f'(.237) = \boxed{1}$

$y - 0.115 = 1(x - 0.237)$
$y - 0.115 = x - 0.237$
$y = x - 0.122$
(slope intercept form)

4. An equation of the line tangent to the graph of y = cos(2x) at $x = \frac{\pi}{4}$ is

All answers for #4 are in point-slope form

(A) y - 1 = -(x - π/4) (B) y - 1 = -2(x - π/4) (C) y = 2(x - π/4)

(D) y = -(x - π/4) (E) y = -2(x - π/4)

Pt: $\boxed{(\pi/4, 0)}$

Slope: $y' = -\sin(2x) \cdot 2$

$y'(\pi/4) = \boxed{-2}$

$y - 0 = -2(x - \pi/4)$
$y = -2(x - \pi/4)$
(point-slope form)

5. At what point on the graph of $y = \frac{1}{2}x^2$ is the tangent line parallel to the line 2x - 4y = 3?

rewrite: $y = \boxed{\frac{1}{2}}x - \frac{3}{4}$

(A) (1/2,-1/2) (B) (1/2,1/8) (C) (1, -1/4)

(D) (1, 1/2) (E) (2, 2)

in other words... WHERE does the graph have a SLOPE of $\frac{1}{2}$... (derivative)

$y = \frac{1}{2}x^2$

$y' = x = \frac{1}{2} \longrightarrow \left(\frac{1}{2}, y\left(\frac{1}{2}\right)\right) = \left(\frac{1}{2}, \frac{1}{8}\right)$

6. An equation of the line tangent to the graph of $f(x) = x(1-2x)^3$ at the point (1, -1) is

All answers for #6 are in slope-intercept form

(A) y = -7x + 6 (B) y = -6x + 5 (C) y = -2x + 1

(D) y = 2x - 3 (E) y = 7x -8

point: $\boxed{(1,-1)}$

Slope: $f'(x) = (1-2x)^3(1) + (x)\cdot 3(1-2x)^2\cdot -2$

$f'(1) = \boxed{-7}$

$y+1 = -7(x-1)$

$y+1 = -7x+7$

$y = -7x+6$

(slope-intercept form)

7. The slope of the line normal to the graph of y = 2 ln(sec x) at $x = \frac{\pi}{4}$ is

(A) -2

(B) $-\frac{1}{2}$

(C) $\frac{1}{2}$

(D) 2

(E) nonexistent

$y' = 2\cdot\frac{\cancel{\sec x}\tan x}{\cancel{\sec x}} = 2\tan x$

$y'(\pi/4) = \boxed{2} \leftarrow$ slope of tangent line

So... $\boxed{-1/2} \leftarrow$ slope of normal line



8. Let $f$ be a differentiable function such that f(3) = 2 and f '(3) = 5. If the tangent line to the graph of f at x = 3 is used to find an approximation to a zero of $f$, that approximation is

(A) 0.4 (B) 0.5 (C) 2.6 (D) 3.4 (E) 5.5

pt: $\boxed{(3,2)}$

Slope: $f'(3) = \boxed{5}$

$y - 2 = 5(x-3)$

$y = 5x - 15 + 2$

$y = 5x - 13 = 0$

$5x - 13 = 0$

$5x = 13$

$x = \frac{13}{5}$

$x = 2\frac{3}{5}$

$\boxed{x = 2.6}$