P. 281 (29-43) odd

29–43 Find the critical numbers of the function.

29. $f(x) = 4 + \frac{1}{3}x - \frac{1}{2}x^2$

31. $f(x) = 2x^3 - 3x^2 - 36x$

33. $g(t) = t^4 + t^3 + t^2 + 1$

35. $g(y) = \dfrac{y-1}{y^2 - y + 1}$

37. $h(t) = t^{3/4} - 2t^{1/4}$

39. $F(x) = x^{4/5}(x-4)^2$

41. $f(\theta) = 2\cos\theta + \sin^2\theta$

43. $f(x) = x^2 e^{-3x}$

29. $y' = \frac{1}{3} - x = 0$

$-x = -\frac{1}{3}$

$x = \frac{1}{3}$

31. $f' = 6x^2 - 6x - 36 = 0$

$6(x^2 - x - 6) = 0$

$6(x-3)(x+2) = 0$

$x = 3, -2$

33. $g' = 4t^3 + 3t^2 + 2t = 0$

$t(4t^2 + 3t + 2) = 0$

$t(\cancel{2t+2})(2t+1) = 0$ (no real roots)

$t = 0$

regular: $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

35. $y' = \dfrac{(y^2 - y + 1)(1) - (y-1)(2y-1)}{(y^2 - y + 1)^2} = 0 \rightarrow \dfrac{y^2 - y + 1 - 2y^2 + 2y + y - 1}{(y^2 - y + 1)^2}$

$= \dfrac{-y^2 + 2y}{(y^2 - y + 1)^2} = 0$

$-y^2 + 2y = 0$

$-y(y-2) = 0$

$y = 0, 2$

$(y^2 - y - 1)^2 = 0$

never

37. $h' = \frac{3}{4}t^{-1/4} - \frac{1}{2}t^{-3/4} = 0$

FACTOR

$-\frac{1}{2}t^{-1/4}\left(\frac{3}{2} + t^{-1/2}\right) = 0$

$t = 0$

$\frac{3}{2} = \frac{1}{t^{1/2}}$

$3t^{1/2} = 2$

$t^{1/2} = 2/3$

$t = 4/9$

*short way

$\frac{3}{4}t^{-1/4} = \frac{1}{2}t^{-3/4}$

$3t^{-1/4} = 2t^{-3/4}$

$\frac{3}{t^{1/4}} = \frac{2}{t^{3/4}}$

$3t^{3/4} = 2t^{1/4}$

$3t^{3/4} - 2t^{1/4} = 0$

$t^{1/4}(3t^{1/2} - 2) = 0$

$t^{1/4} = 0$

$t = 0$

$3t^{1/2} - 2 = 0$

$3t^{1/2} = 2$

$t^{1/2} = 2/3$

$t = (2/3)^2 = \frac{4}{9}$

x long way

39. $F(x) = (x^{4/5})(x-4)^2$

$F'(x) = (x-4)^2 \cdot \frac{4}{5}x^{-1/5} + x^{4/5}(2(x-4)) = 0$

$\frac{1}{5}x^{-1/5}(x-4)\left[4(x-4) + 2 \cdot 5 \cdot x\right] = 0$

$\frac{1}{5}x^{-1/5} = 0$ (never)

$x^{-1/5} = 0$

$\frac{1}{x^{1/5}} = 0$

$x = 0$ where $f'(x)$ = DNE

$x - 4 = 0$

$x = 4$ where $f'(x) = 0$

$4(x-4) + 10x = 0$

$4x - 16 + 10x = 0$

$14x = 16$

$x = 16/14 = 8/7$ where $f'(x) = 0$

41. $f' = -2\sin\theta + 2\sin\theta\cos\theta = 0$

$$\frac{2\sin\theta\cos\theta}{2\sin\theta} = \frac{2\sin\theta}{2\sin\theta}$$

where numerator = 0, $f'(x) = 0$ $\rightarrow \cos\theta = 1$

$\theta = 0, 2\pi, 4\pi$ $\rightarrow 2n\pi$

where denom. = 0, $f'(x)$ = DNE $\rightarrow \sin\theta = 0$

$\theta = 0, 2\pi, 4\pi$

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43. $f' = (x^2)(-3e^{-3x}) + (e^{-3x})(2x) = 0$

FACTOR $xe^{-3x}(-3x + 2) = 0$

$xe^{-3x}(3x + 2) = 0$

$x = 0$ $\quad x = 2/3$
