Calculus Name_____ ID: 1

Implicit Differentiation

Date_____ Period____

For each problem, use implicit differentiation to find $\frac{dy}{dx}$ in terms of x and y.

1) $-2y^2 + 4 = 5x$

$-4y\frac{dy}{dx} = 5$

$\frac{dy}{dx} = -\frac{5}{4y}$

2) $3x^3 - y^2 = 5$

$9x^2 - 2y\frac{dy}{dx} = 0$

$-2y\frac{dy}{dx} = -9x^2$

$\frac{dy}{dx} = \frac{9x^2}{2y}$

3) $5x^3 = -4y^2 + 2$

$15x^2 = -8y\frac{dy}{dx}$

$\frac{dy}{dx} = \frac{15x^2}{-8y}$

4) $y^3 + 4 = 3x^3$

$3y^2\frac{dy}{dx} = 9x^2$

$\frac{dy}{dx} = \frac{9x^2}{3y^2} = \frac{3x^2}{y^2}$

5) $4 = x^3(-5x^2)(y^3)$

$0 = 3x^2 + (y^3)(-10x) + (-5x^2)(3y^2\frac{dy}{dx})$

$0 = 3x^2 - 10xy^3 - 15x^2y^2\frac{dy}{dx}$

$-3x^2 + 10xy^3 = -15x^2y^2\frac{dy}{dx}$

$\frac{dy}{dx} = \frac{-3x^2 + 10xy^3}{-15x^2y^2}$

6) $(-3x^2)(y) + 2 = 4x^2$

$(y)(-6x) + (-3x^2)(\frac{dy}{dx}) = 8x$

$-3x^2\frac{dy}{dx} = 8x + 6xy$

$\frac{dy}{dx} = \frac{8x + 6xy}{-3x^2}$

For each problem, use implicit differentiation to find $\frac{dy}{dx}$ at the given point.

7) $-3y^3 + 4 = x^2$ at $(1, 1)$

$-9y^2\frac{dy}{dx} = 2x$

$\frac{dy}{dx} = \frac{2x}{-9y^2}$

$\left.\frac{dy}{dx}\right|_{(1,1)} = \frac{2(1)}{-9(1)^2} = -\frac{2}{9}$

8) $-y^3 + 1 = x$ at $(2, -1)$

$-3y^2\frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{-1}{3y^2}$

$\left.\frac{dy}{dx}\right|_{(2,-1)} = \frac{-1}{3(-1)^2} = -\frac{1}{3}$

9) $4x = -5y^2 + 1$ at $(-1, 1)$

$4 = -10y\frac{dy}{dx}$

$\frac{dy}{dx} = \frac{4}{-10y}$

$\left.\frac{dy}{dx}\right|_{(1,1)} = \frac{4}{-10(1)} = -\frac{4}{10} = -\frac{2}{5}$

10) $2x^3 = (-x^3)(y^3) + 3$ at $(1, 1)$

$6x^2 = (y^3)(-3x^2) + (-x^3)(3y^2\frac{dy}{dx})$

$6x^2 + 3y^3x^2 = -3x^3y^2\frac{dy}{dx}$

$\frac{dy}{dx} = \frac{6x^2 + 3y^3x^2}{-3x^3y^2}$

$\left.\frac{dy}{dx}\right|_{(1,1)} = \frac{6(1)^2 + 3(1)^3(1)^2}{-3(1)^3(1)^2} = \frac{9}{-3} = -3$

For each problem, use implicit differentiation to find $\frac{dy}{dx}$ in terms of x and y.

11) $3x^3 + 5y^3 = 5y$

$9x^2 + 15y^2 \frac{dy}{dx} = 5\frac{dy}{dx}$

$9x^2 = 5\frac{dy}{dx} - 15y^2\frac{dy}{dx}$

$9x^2 = (5 - 15y^2)\frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{9x^2}{5 - 15y^2}$$

12) $4x + 2y = 4y^3$

$4 + 2\frac{dy}{dx} = 12y^2\frac{dy}{dx}$

$4 = 12y^2\frac{dy}{dx} - 2\frac{dy}{dx}$

$4 = (12y^2 - 2)\frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{4}{12y^2 - 2}$$

For each problem, use implicit differentiation to find $\frac{d^2y}{dx^2}$ in terms of x and y.

13) $4x = 2y^2 + 1$

$4 = 4y\frac{dy}{dx} + 0$

$\frac{dy}{dx} = \frac{4}{4y}$

$$\frac{dy}{dx} = \frac{1}{y}$$

$\frac{d^2y}{dx^2} = \frac{(y)(0) - (1)(\frac{dy}{dx})}{y^2}$

$\frac{d^2y}{dx^2} = \frac{-1/y}{y^2}$

$$\frac{d^2y}{dx^2} = \frac{-1}{y^3}$$

14) $-4y^2 + 2 = 4x$

$-8y\frac{dy}{dx} + 0 = 4$

$\frac{dy}{dx} = \frac{4}{-8y}$

$$\frac{dy}{dx} = -\frac{1}{2y}$$

$\frac{d^2y}{dx^2} = \frac{(2y)(0) - (-1)(2\frac{dy}{dx})}{(2y)^2}$

$\frac{d^2y}{dx^2} = \frac{2(-1/2y)}{(2y^2)}$

$\frac{d^2y}{dx^2} = \frac{-1/y}{4y^2}$

$$\frac{d^2y}{dx^2} = -\frac{1}{4y^3}$$