U4H4

1. $y = 2x^4 - 4x^2 + 1$

Critical Points	$x = 0, \pm 1$
Intervals of Inc.	$(-1, 0)$ $(1, \infty)$
Intervals of Dec.	$(-\infty, -1)$ $(0, 1)$
Inflection Points	$y = \pm\sqrt{1/3}$
Concave Up	$(-\infty, -\sqrt{1/3})$ $(\sqrt{1/3}, \infty)$
Concave Down	$(-\sqrt{1/3}, \sqrt{1/3})$
Extrema	rel max: $x = 0$ abs min: $x = \pm 1$

$y' = 8x^3 - 8x = 0$

$8x(x^2 - 1) = 0$

$x = 0, \pm 1$

y': $-$ | -1 | $+$ | 0 | $-$ | 1 | $+$

$y'' = 24x^2 - 8 = 0$

$8(3x^2 - 1) = 0$

$x = \pm\sqrt{1/3}$

y'': $+$ | $-\sqrt{1/3}$ | $-$ | $\sqrt{1/3}$ | $+$

2. $y = x^4 - 8x^2 + 1$

Critical Points	$x = 0, \pm 2$
Intervals of Inc.	$(-2, 0)$ $(2, \infty)$
Intervals of Dec.	$(-\infty, -2)$ $(0, 2)$
Inflection Points	$x = \pm\sqrt{4/3}$
Concave Up	$(-\infty, -\sqrt{4/3})$ $(\sqrt{4/3}, \infty)$
Concave Down	$(-\sqrt{4/3}, \sqrt{4/3})$
Extrema	rel max: $x = 0$ abs min: $x = \pm 2$

$y' = 4x^3 - 16x = 0$

$4x(x^2 - 4) = 0$

$x = 0, \pm 2$

y': $-$ | -2 | $+$ | 0 | $-$ | 2 | $+$

$y'' = 12x^2 - 16 = 0$

$4(3x^2 - 4) = 0$

$x = \pm\sqrt{4/3}$

y'': $+$ | $-\sqrt{4/3}$ | $-$ | $\sqrt{4/3}$ | $+$

Use the Concavity Test to determine the intervals on which the graph of the function is concave up and concave down.

3. $y = 2x^{1/5} + 3$

$y' = \frac{2}{5}x^{-4/5}$

$y'' = -\frac{8}{25}x^{-9/5} = 0$

$x = 0$

(where y'' DNE)

y'': $+$ | 0 | $-$

con. up: $(-\infty, 0)$
con. down: $(0, \infty)$

4. $y = 5 - x^{1/3}$

$y' = -\frac{1}{3}x^{-2/3}$

$y'' = \frac{2}{9}x^{-5/3} = 0$

$x = 0$

(where y'' DNE)

y'': $-$ | 0 | $+$

con. up: $(0, \infty)$
con. down: $(-\infty, 0)$

Find all points of inflection of the function.

5. $y = xe^x$

$y' = (e^x)(1) + (x)(e^x)$

$y'' = e^x + e^x + xe^x = 0$

$2e^x + xe^x = 0$

$e^x(2 + x) = 0$

none $\quad$ $x = -2$

y'': $-$ | -2 | $+$

$\boxed{x = -2}$

6. $y = \tan^{-1} x$

$y' = \frac{1}{1 + x^2}$

$y'' = \frac{(1+x^2)(0) - (1)(2x)}{(1+x^2)^2} = 0$

$\frac{-2x}{(1+x^2)^2} = 0$

$x = 0$

y'': $+$ | 0 | $-$

$\boxed{x = 0}$

7. $y = 3x - x^3 + 5$

a. Use the first derivative test to find the local extrema for the function.

$y' = 3 - 3x^2 = 0$

$3x^2 = 3$

$x^2 = 1$

$x = \pm 1$

y': $-$ | -1 | $+$ | 1 | $-$

rel max: $x = 1$

rel min: $x = -1$

b. Use the second derivative test to find the local extrema for the function.

$y' = 3 - 3x^2$

critical pts: $x = \pm 1$

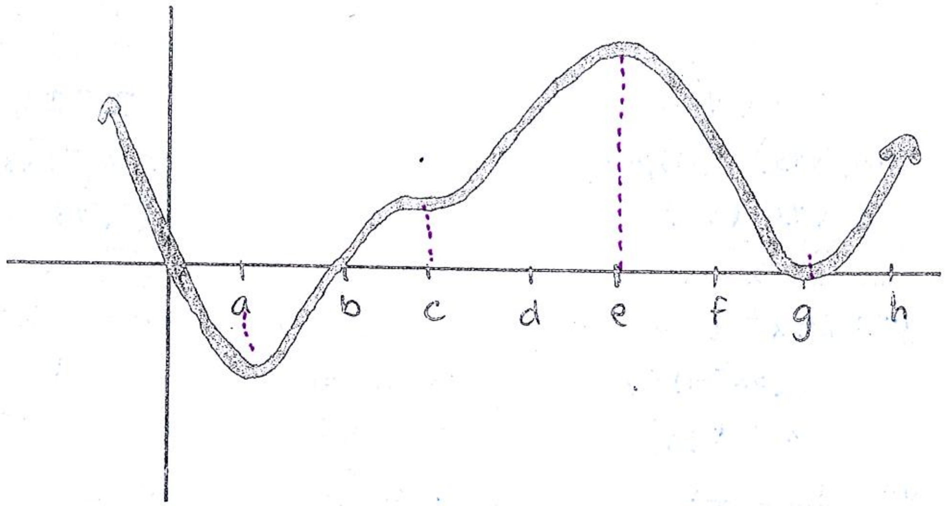
$y'' = -6x$

$y''(-1) > 0 \leftarrow$ concave up

$y''(1) < 0 \leftarrow$ concave down

rel max: $x = 1$

rel min: $x = -1$



If graph is $f(x)$, name all x-values where $f(x)$ has:

- critical point: $x = a, c, e, g$

If graph is $f'(x)$, name all x-values where $f(x)$ has:

- critical point: $x = 0, b, g$
- minimum: $x = b$
- maximum: $x = 0$
- interval of increasing: $(-\infty, 0)$ (b, ∞)
- interval of decreasing: $(0, b)$
- inflection points: $x = a, e, g$
- interval of concove up: (a, e) (g, ∞)
- interval of concave down: $(-\infty, a)$ (e, g)

If graph is $f''(x)$, name all x-values where $f(x)$ has

- inflection points: $x = 0, b$
- interval of convcave up: $(-\infty, 0)$ (b, ∞)
- interval of concave down: $(0, b)$