Calculus Name______________________________ ID: 1

Curve Sketching

Date________________ Period____

For each problem, find the: x and y intercepts, x-coordinates of the critical points, open intervals where the function is increasing and decreasing, x-coordinates of the inflection points, open intervals where the function is concave up and concave down, and relative minima and maxima. Using this information, sketch the graph of the function.

✓ 1) $y = 2x^3 + 4x^2 + 2x$

x-int:

$0 = 2x^3 + 4x^2 + 2x$

$0 = 2x(x^2 + 2x + 1)$

$0 = 2x(x+1)^2$

$x = 0, -1$

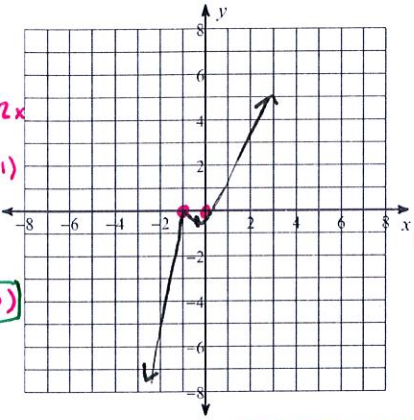
$\boxed{(0,0) + (-1,0)}$

y-int:

$y = 0 + 0 + 0$

$= 0$

$\boxed{(0,0)}$



$y' = 6x^2 + 8x + 2 = 0/DNE$

$2(3x^2 + 4x + 1) = 0$

$2(3x+1)(x+1) = 0$

$x = -1/3, -1$

y' + ∩ − ∪ +

−1 $-\frac{1}{3}$

$y'' = 12x + 8 = 0/DNE$

$4(3x+2) = 0$

$x = -2/3$

y'' − +

$-\frac{2}{3}$

critical pt(s): $x = -1/3$ + -1

int. of inc: $(-\infty, -\frac{1}{3}) \cup (-1, \infty)$

int. of dec: $(-\frac{1}{3}, 1)$

extrema:

$x = -1/3$ rel min

$x = -1$ rel max

infl. pt(s): $x = -2/3$

int. con. up: $(-2/3, \infty)$

int. con. down: $(-\infty, -2/3)$

× 2) $\xcancel{y = -\frac{x^3}{x^2 - 1}}$

OMIT

x-int:

$0 = -\frac{x^3}{x^2-1}$

$0 = -x^3$

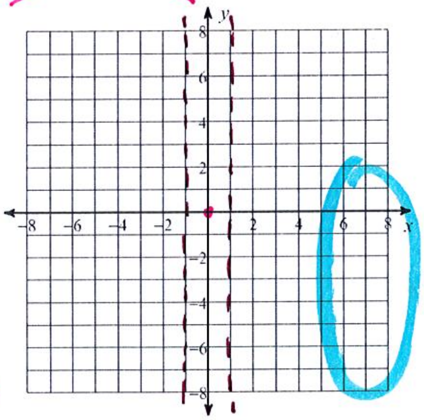
$x = 0$

$\boxed{(0,0)}$

y-int:

$y = \frac{-0^3}{0^2-1} = 0$

$\boxed{(0,0)}$



$y' = \frac{(x^2-1)(-3x^2) - (-x^3)(2x)}{(x^2-1)^2} = 0/DNE$

$\frac{-3x^4 + 3x^2 + 2x^4}{(x^2-1)^2} = 0/DNE$

$\frac{-x^4 + 3x^2}{(x^2-1)^2} = 0/DNE$

$\frac{-x^2(x^2-3)}{(x^2-1)^2} = 0$

$x = 0, \pm\sqrt{3}, \xcancel{\pm 1}$ not in domain

y'

$-\sqrt{3}$ 0 $\sqrt{3}$

$y'' = \frac{(x^2-1)^2(-4x^3+6x) - (-x^4+3x^2)(2(x^2-1)\cdot 2x)}{(x^2-1)^4} =$

$\frac{(x^2-1)\left[(x^2-1)(-4x^3+6x) - (-x^4+3x^2)(4x)\right]}{(x^2-1)^4}$

x 3) ~~$f(x) = \frac{x^3}{12} - \frac{x^2}{6} - \frac{x}{3}$~~

x-int:

$0 = \frac{x^3}{12} - \frac{x^2}{6} - \frac{x}{3}$

$0 = x^3 - 2x^2 - 4x$

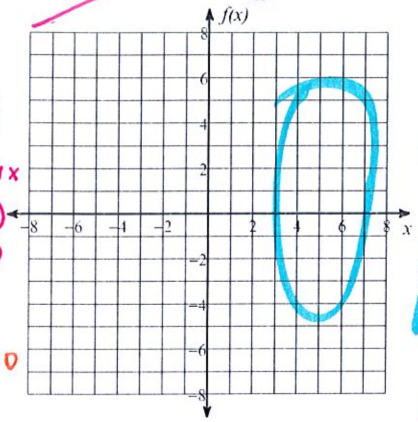
$= x(x^2 - 2x - 4)$

$= x($~~$x - 2$~~$)$

y-int:

$y = 0^3 - x^2 - 0 = 0$

$(0, 0)$



$f' = \frac{1}{4}x^2 - \frac{1}{3}x - \frac{1}{3} = 0/DNE$

$3x^2 - 4x - 4 = 0$

$(3x + 2)(x - 2) = 0$

$x = -2/3,\ 2$

f' ←—|—|—→ $-\frac{2}{3}$, 2

$f'' = 6x - 4 = 0/DNE$

$2(3x - 2) = 0$

$x = 2/3$

f'' ←— − —|— + —→ $\frac{2}{3}$

OMIT

✓4) $f(x) = -(5x + 30)^{\frac{1}{2}}$

x-int:

$= -(5x+30)^{1/2}$

$= (5x+30)^{1/2}$

$= 5x + 30$

$30 = 5x$

$x = -6$

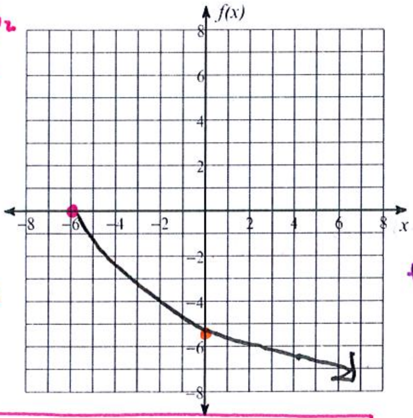
$\boxed{(-6, 0)}$

y-int:

$= -(0+30)^{1/2}$

$= -\sqrt{30}$

$\boxed{(0, -\sqrt{30})}$



$f' = -\frac{1}{2}(5x+30)^{-1/2} \cdot 5 = 0/DNE$

$\frac{-5}{2\sqrt{5x+30}} = 0/DNE$

$x = 6$

f' ←— DNE —|— − —→ −6

$f'' = \frac{5}{2}(5x+30)^{-3/2} \cdot 5 = 0/DNE$

$\frac{25}{2(5x+30)^{-3/2}} = 0/DNE$

$x = -6$

f'' ←— DNE —|— + —→ −6

critical pt(s): $x = -6$

int. of inc: none

int. of dec: $(-6, \infty)$

extrema: none

infl. pt(s): none

int. con. up: $(-6, \infty)$

int. con. down: none

✓5) $y = -x^3 - 3x^2$

x-int:

$0 = -x^3 - 3x^2$

$0 = -x^2(x+3)$

$x = 0, -3$

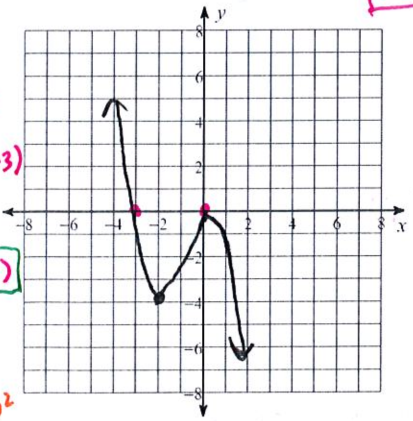
$\boxed{(0,0) + (-3, 0)}$

y-int:

$y = -0^3 - 3(0)^2$

$y = 0$

$\boxed{(0, 0)}$



$f' = -3x^2 - 6x = 0/DNE$

$-3x(x + 2) = 0$

$x = 0, -2$

f' ←— − ∪ —|— + ∩ —|— − —→ −2, 0

$f'' = -6x - 6 = 0/DNE$

$-6(x + 1) = 0$

$x = -1$

f'' ←— + —|— − —→ −1

critical pt(s): $x = -2, 0$

int. of inc: $(-2, 0)$

int. of dec: $(-\infty, -2) \cup (0, \infty)$

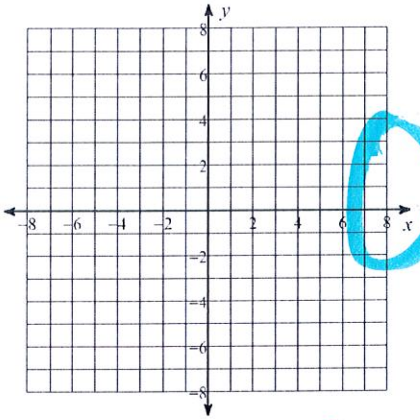
extrema:
$x = -2$ rel min
$x = 0$ rel max

infl. pt(s): $x = -1$

int. con. up: $(-\infty, -1)$

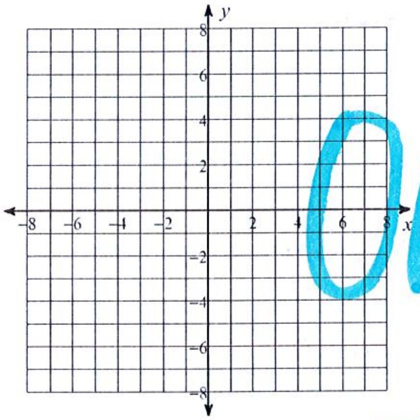
int. con. down: $(-1, \infty)$

✗ 6) ~~$y = -\frac{3}{16}(x+2)^{\frac{4}{3}} + \frac{3}{2}(x+2)^{\frac{1}{3}}$~~



OMIT

✗ 7) ~~$y = \frac{3}{16}(x-2)^{\frac{4}{3}} + \frac{3}{2}(x-2)^{\frac{1}{3}}$~~



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✓ 8) $f(x) = (-x+5)^{\frac{1}{2}}$

x-int:

$0 = (-x+5)^{1/2}$

$0 = -x+5$

$x = 5$

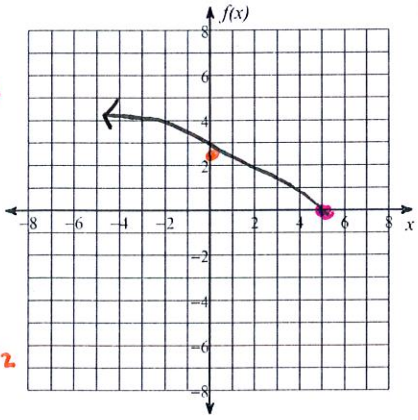
$\boxed{(5,0)}$

y-int:

$y = (-0+5)^{1/2}$

$y = \sqrt{5}$

$\boxed{(0,\sqrt{5})}$



$f' = \frac{1}{2}(-x+5)^{-1/2} \cdot -1 = 0/DNE$

$-\frac{1}{2} \cdot \frac{1}{\sqrt{-x+5}} = 0/DNE$

$x = 5$

f' − DNE

5

$f'' = -\frac{1}{2} \cdot -\frac{1}{2}(-x+5)^{-3/2} \cdot -1 = 0/DN$

$-\frac{1}{4}(-x+5)^{-3/2} = 0/DNE$

$-\frac{1}{4} \cdot \frac{1}{\sqrt{(-x+5)^3}} = 0/DNE$

$x = 5$

f'' − DNE

5

critical pt(s):	$x=5$
int. of inc:	none
int. of dec:	$(-\infty, 5)$
extrema:	none
infl. pt(s):	none
int. con. up:	none
int. con. down:	$(-\infty, 5)$

✓9) $f(x) = -\frac{x^3}{12} - \frac{x^2}{4}$

x-int:

$= -\frac{x^3}{12} - \frac{x^2}{4}$

$0 = -x^3 - 3x^2$

$0 = -x^2(x+3)$

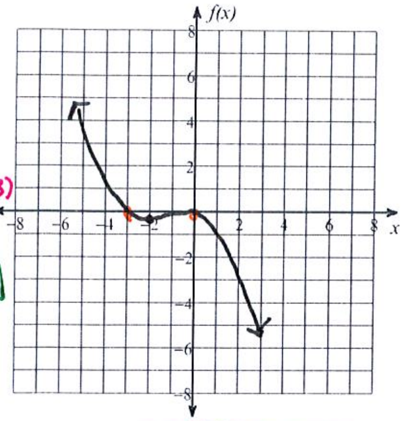
$x = 0, -3$

$\boxed{(0,0) + (-3,0)}$

-int:

$y = 0 - 0 = 0$

$\boxed{(0,0)}$

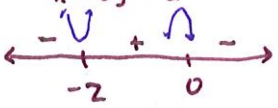


$f' = -\frac{1}{4}x^2 - \frac{1}{2}x = 0/DNE$

$-\frac{1}{4}x(x+2) = 0$

$x = 0, -2$

f'



$f'' = -\frac{1}{2}x - \frac{1}{2} = 0/DNE$

$-\frac{1}{2}(x+1) = 0$

$x = -1$

f''

critical pt(s):	$x = -2, 0$
int. of inc:	$(-2, 0)$
int. of dec:	$(-\infty, -2) \cup (0, \infty)$
extrema:	$x = -2$ rel min $x = 0$ rel max
infl. pt(s):	$x = -1$
int. con. up:	$(-\infty, -1)$
int. con. down:	$(-1, \infty)$

✓10) $f(x) = -(5x + 5)^{\frac{1}{3}}$

x-int:

$0 = -(5x+5)^{1/3}$

$0 = 5x+5$

$-5 = 5x$

$x = -1$

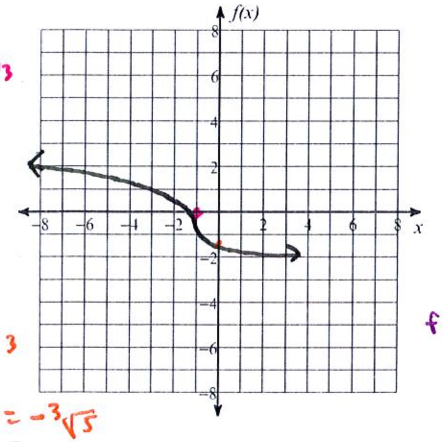
$\boxed{(-1,0)}$

y-int:

$= -(0+5)^{1/3}$

$y = -(5)^{1/3} = -\sqrt[3]{5}$

$\boxed{(0, -\sqrt[3]{5})}$



$f' = -\frac{1}{3}(5x+5)^{-2/3} \cdot 5 = 0/DNE$

$\frac{-5}{3} \cdot \frac{1}{(5x+5)^{2/3}} = 0/DNE$

$x = -1$

f'

$f'' = -\frac{5}{3} \cdot -\frac{2}{3}(5x+5)^{-5/3} = 0/DNE$

$\frac{10}{3} \cdot \frac{1}{(5x+5)^{5/3}} = 0/DNE$

$x = -1$

f''

critical pt(s):	$x = -1$
int. of inc:	none
int. of dec:	$(-\infty, \infty)$
extrema:	none
infl. pt(s):	$x = -1$
int. con. up:	$(-1, \infty)$
int. con. down:	$(-\infty, -1)$