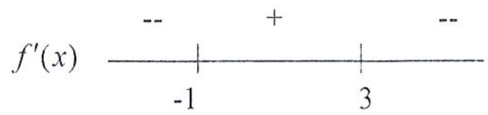
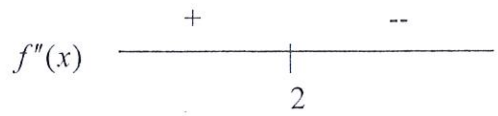
CRITICAL POINTS - PART 2

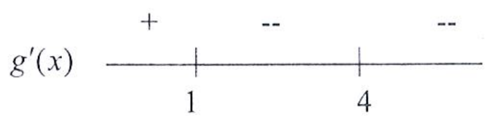
1. In each case, sketch a graph of a continuous function with the given properties.

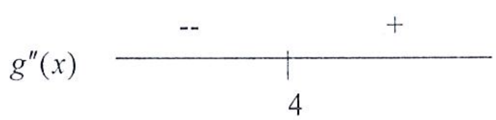
A. $f'(-1)=0$ and $f'(3)=0$



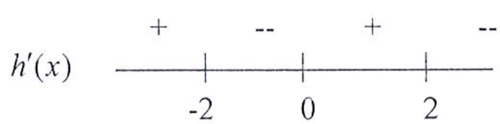


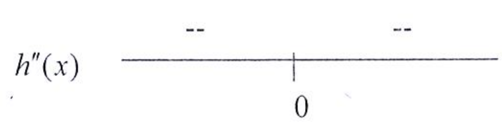
B. $g'(1)=0$ and $g'(4)$ is undefined





C. $h'(-2)=0$ and $h'(2)=0$
$h'(0)$ is undefined





2. Use Calculus to determine i) critical points, ii) local extrema, iii) inflection points, and iv) intervals where $f(x)$ is concave up or down. Include an accurate graph that illustrates these features. Do this on a separate sheet of paper.

A. $f(x)=x^4+2x^3-1$ B. $f(x)=\dfrac{8x-16}{x^2}$ C. $f(x)=2x+3x^{2/3}$

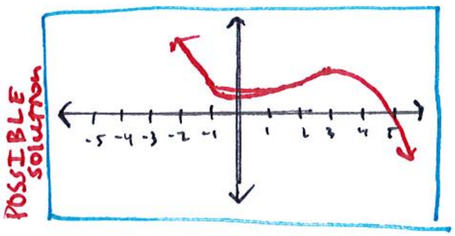
Curve Sketching

(on whiteboards) - draw graph + label $-5 \le x \le 5$)

1. $f'(-1) = 0$
   $f'(3) = 0$

$f'(x)$: $-$ | -1 | $+$ | 3 | $-$

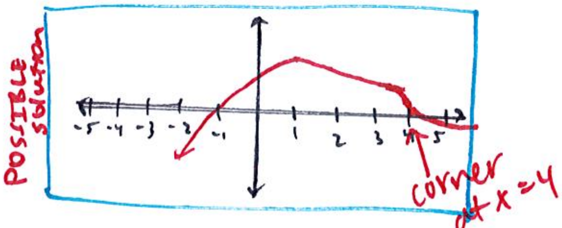
$f''(x)$: $+$ | 2 | $-$



2. $g'(1) = 0$
   $g'(4) = \text{DNE}$

$g'(x)$: $+$ | 1 | $-$ | 4 | $-$

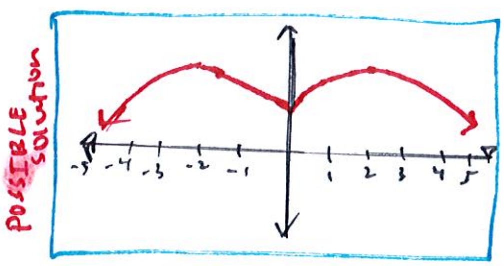
$g''(x)$: $-$ | 4 | $+$



3. $h'(-2) = 0$
   $h'(2) = 0$
   $h'(0) = \text{DNE}$

$h'(x)$: $+$ | -2 | $-$ | 0 | $+$ | 2 | $-$

$h''(x)$: $-$ | 0 | $-$



A. $\boxed{f(x) = x^4 + 2x^3 - 1}$

$f'(x) = 4x^3 + 6x^2 = 0$

$2x^2(2x+3) = 0$

$\boxed{x = 0, -3/2}$ → critical points

$f'(x)$: $-$ | $-3/2$ | $+$ | 0 | $+$

extrema

$\boxed{x = -3/2 \text{ abs min}}$

$f''(x) = 12x^2 + 12x = 0$

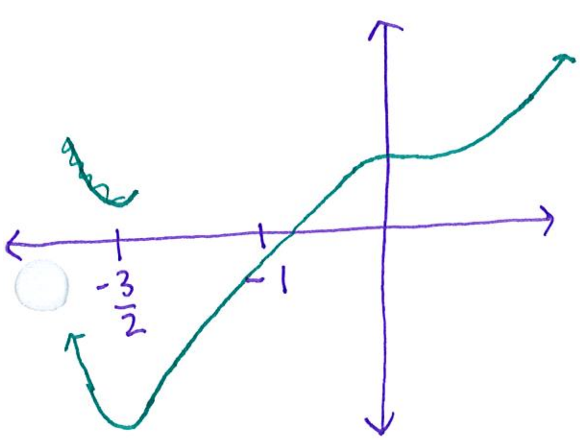
$12x(x+1) = 0$

$\boxed{x = 0, -1}$ → inflection points

$f''(x) =$ $-\infty$ | $+$ | -1 | $-$ | 0 | $+$ | ∞

concave up:
$(-\infty, -1)$ $(0, \infty)$

concave down:
$(-1, 0)$



graph of $f(x)$

B. $\boxed{f(x) = \frac{8x-16}{x^2}} = \frac{8}{x} - \frac{16}{x^2} = 8x^{-1} - 16x^{-2}$

note $f(x)$ has a VA at $x=0$

$f'(x) = -8x^{-2} + 32x^{-3} = 0$

$$-\frac{8}{x^2} + \frac{32}{x^3} = 0$$

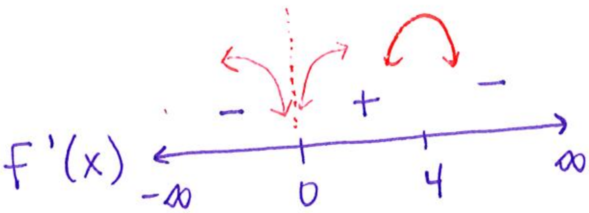
$$\frac{32}{x^3} = \frac{8}{x^2}$$

$$32x^2 = 8x^3$$

$$32x^2 - 8x^3 = 0$$

$$8x^2(4-x) = 0$$

$\boxed{x = 0, 4}$ → critical points



$\boxed{x=4 \quad \text{abs max}}$ → extrema

$f''(x) = 16x^{-3} - 96x^{-4} = 0$

$$\frac{16}{x^3} - \frac{96}{x^4} = 0$$

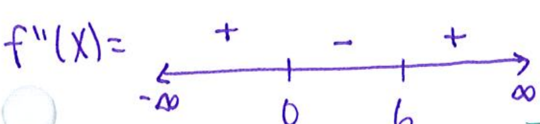
$$\frac{16}{x^3} = \frac{96}{x^4}$$

$$16x^4 = 96x^3$$

$$16x^4 - 96x^3 = 0$$

$$16x^3(x-6) = 0$$

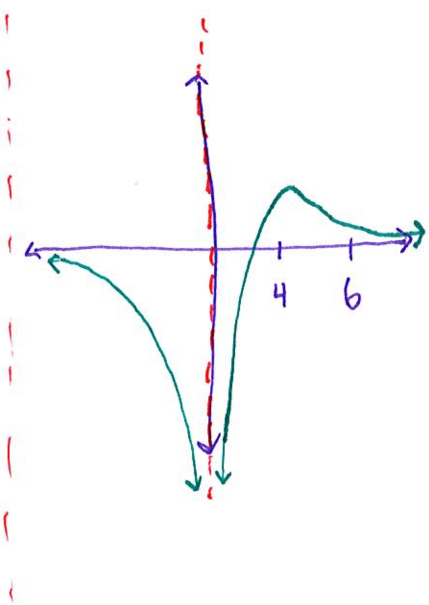
$\boxed{x = 0, 6}$ → inflection points



Concave up: $(-\infty, 0)$ $(6, \infty)$

Concave down: $(0, 6)$

graph of $f(x)$



C. $\boxed{f(x) = 2x + 3x^{2/3}}$

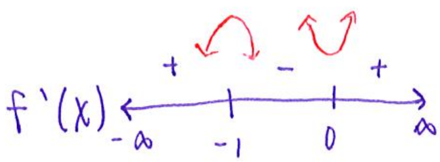
$$f'(x) = 2 + 2x^{-1/3} = 0$$

$$2 + \frac{2}{x^{1/3}} = 0$$

$$2 = -\frac{2}{x^{1/3}}$$

$$2x^{1/3} = -2$$

$$x^{1/3} = -1$$



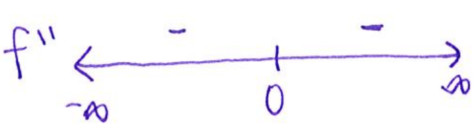
$\boxed{x = 0, -1}$ → critical points

$\boxed{\begin{array}{l} x=-1 \text{ rel max} \\ x=0 \text{ rel min} \end{array}}$ → extrema

$$f''(x) = -\frac{2}{3}x^{-4/3} = 0$$

$$x^{-4/3} = 0$$

$$\frac{1}{x^{4/3}} = 0$$



$\boxed{x = 0}$ → NOT an inflection point

concave down: $(-\infty, \infty)$

graph of $f(x)$