Derivatives AP Exam Practice

$6x + 2y + 2x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{-6x-2y}{2x+2y}$

1.

If $3x^2 + 2xy + y^2 = 2$, then the value of $\frac{dy}{dx}$ at $x = 1$ is (1,-1)

$\left.\frac{dy}{dx}\right|_{(1,\ )} = \frac{-6(1)-2(-1)}{2(1)+2(-1)} = \frac{4}{0} \leftarrow$:(

(A) -2 (B) 0 (C) 2 (D) 4 (E) not defined

2.

If $f(x) = 2 + |x-3|$ for all x, then the value of the derivative $f'(x)$ at $x = 3$ is

since f(3) is a corner

(A) -1 (B) 0 (C) 1 (D) 2 (E) nonexistent

3.

An equation for a tangent to the graph of $y = \arcsin\frac{x}{2}$ at the origin is

pt $\boxed{(0,0)}$

slope $y' = \frac{1}{\sqrt{1-(\frac{x}{2})^2}} \cdot \frac{1}{2}$

(A) $x - 2y = 0$ (B) $x - y = 0$ (C) $x = 0$ (D) $y = 0$ (E) $\pi x - 2y = 0$

$y'(0) = \boxed{\frac{1}{2}}$

equation: $y = \frac{1}{2}x$

4.

$\frac{d}{dx}\left(\ln e^{2x}\right) = \frac{1}{\cancel{e^{2x}}} \cdot \cancel{e^{2x}} \cdot 2 = 2$

(A) $\frac{1}{e^{2x}}$ (B) $\frac{2}{e^{2x}}$ (C) $2x$ (D) 1 (E) 2

5.

no y in answer!!

If $\sin x = e^y$, $0 < x < \pi$, what is $\frac{dy}{dx}$ in terms of x?

$\cos x = e^y \cdot \frac{dy}{dx}$

$\frac{dy}{dx} = \frac{\cos x}{e^y} = \frac{\cos x}{\sin x} = \cot x$

(A) $-\tan x$ (B) $-\cot x$ (C) $\cot x$ (D) $\tan x$ (E) $\csc x$

6.

$f(1) = 1$

If $f'(x) = -f(x)$ and $f(1) = 1$, then $f(x) =$

(A) $\frac{1}{2}e^{-2x+2}$ (B) e^{-x-1} (C) e^{1-x} (D) e^{-x} (E) $-e^x$

$f' = \cancel{\frac{1}{2}}e^{-2x+2} \cdot -\cancel{2}$ $= -e^{-2x+2}$ | $f' = e^{-x-1} \cdot -1$ $= -e^{-x-1}$ | $f' = e^{1-x} \cdot -1$ $= -e^{1-x}$ | $f' = e^{-x} \cdot -1$ $= -e^{-x}$ | $f' = -e^x \cdot 1$ $= -e^x$

7.

If $y = \tan u$, $u = v - \frac{1}{v}$, and $v = \ln x$, what is the value of $\frac{dy}{dx}$ at $x = e$?

REWRITE:

$y = \tan(v - \frac{1}{v})$

$= \tan(\ln x - \frac{1}{\ln x})$

(A) 0 (B) $\frac{1}{e}$ (C) 1 (D) $\frac{2}{e}$ (E) $\sec^2 e$

$y' = \sec^2(\ln x - \frac{1}{\ln x})(\frac{1}{x} + (\ln x)^{-2} \cdot \frac{1}{x})$

$y'(e) = \sec^2(1-1)(\frac{1}{e} + (1)^{-2} \cdot \frac{1}{e})$

$= \sec^2(0) \cdot (\frac{2}{e})$

$= 1 \cdot 2/e$

8.

If $\frac{d}{dx}(f(x)) = g(x)$ and $\frac{d}{dx}(g(x)) = f(x^2)$, then $\frac{d^2}{dx^2}\left(f(x^3)\right) =$

$\frac{d}{dx}(f(x^3)) = f'(x^3) \cdot 3x^2$

$\frac{d^2}{dx^2}(f(x^3)) = 3x^2 \cdot f''(x^3) \cdot 3x^2 + f'(x^3) \cdot 6x$
$= 9x^4 \cdot f''(x^3) + 6x \cdot g(x^3)$
$= 9x^4 \cdot f(x^6) + 6x\, g(x^3)$

(A) $f\left(x^6\right)$ (B) $g\left(x^3\right)$ (C) $3x^2 g\left(x^3\right)$

(D) $9x^4 f\left(x^6\right) + 6x\, g\left(x^3\right)$ (E) $f\left(x^6\right) + g\left(x^3\right)$

9.

The slope of the line tangent to the graph of $y = \ln\left(x^2\right)$ at $x = e^2$ is

$y = \ln x^2$
$y' = \frac{1}{x^2} \cdot 2x = \frac{2}{x}$
$y'(e^2) = \frac{2}{e^2}$

(A) $\frac{1}{e^2}$ (B) $\frac{2}{e^2}$ (C) $\frac{4}{e^2}$ (D) $\frac{1}{e^4}$ (E) $\frac{4}{e^4}$

10.

If $f(x) = x + \sin x$, then $f'(x) =$

$f'(x) = 1 + \cos x$

(A) $1 + \cos x$ (B) $1 - \cos x$ (C) $\cos x$
(D) $\sin x - x\cos x$ (E) $\sin x + x\cos x$

11.

If $f(x) = \frac{x-1}{x+1}$ for all $x \neq -1$, then $f'(1) =$

$f'(x) = \frac{(x+1) - (x-1)}{(x+1)^2}$

(A) -1 (B) $-\frac{1}{2}$ (C) 0 (D) $\frac{1}{2}$ (E) 1

$f'(1) = \frac{(2) - (0)}{(2)^2} = \frac{2}{4} = \frac{1}{2}$

12.

If $y = \cos^2 3x$, then $\frac{dy}{dx} =$

REWRITE y

$y = (\cos 3x)^2$
$y' = 2(\cos 3x)' \cdot -\sin 3x \cdot 3$
$= -6 \cos 3x \sin 3x$

(A) $-6\sin 3x \cos 3x$ (B) $-2\cos 3x$ (C) $2\cos 3x$
(D) $6\cos 3x$ (E) $2\sin 3x \cos 3x$

13.

If the line $3x - 4y = 0$ is tangent in the first quadrant to the curve $y = x^3 + k$, then k is

1 2

$y = \boxed{\frac{3}{4}}x$

slope

(A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) 0 (D) $-\frac{1}{8}$ (E) $-\frac{1}{2}$

$y = x^3 + k$
$y' = 3x^2 = \frac{3}{4}$
$x^2 = \frac{1}{4}$
$x = \frac{1}{2}$

sub 1: $3(\frac{1}{2}) - 4y = 0$
$y = 3/8$
sub 2: $\frac{3}{8} = (\frac{1}{2})^3 + k$

14.

$\frac{d}{dx}(\arcsin 2x) =$ $\frac{1}{\sqrt{1-(2x)^2}} \cdot 2 = \frac{2}{\sqrt{1-4x^2}}$

(A) $\frac{-1}{2\sqrt{1-4x^2}}$ (B) $\frac{-2}{\sqrt{4x^2-1}}$ (C) $\frac{1}{2\sqrt{1-4x^2}}$

(D) $\frac{2}{\sqrt{1-4x^2}}$ (E) $\frac{2}{\sqrt{4x^2-1}}$

15.

If $y = e^{nx}$, then $\frac{d^n y}{dx^n} =$

(A) $n^n e^{nx}$ (B) $n!e^{nx}$ (C) $n e^{nx}$ (D) $n^n e^x$ (E) $n!e^x$

$\frac{dy}{dx} = e^{nx} \cdot n = n^1 e^{nx}$

$\frac{d^2y}{dx^2} = ne^{nx} \cdot n = n^2 e^{nx}$

note pattern

$\frac{d^n y}{dx^n} = n^n e^{nx}$

16.

If $\tan(xy) = x$, then $\frac{dy}{dx} =$

$\sec^2(xy) \cdot (y + x\frac{dy}{dx}) = 1$

$y + x\frac{dy}{dx} = \cos^2(xy)$

$\frac{dy}{dx} = \frac{\cos^2(xy) - y}{x}$

(A) $\frac{1 - y\tan(xy)\sec(xy)}{x\tan(xy)\sec(xy)}$ (B) $\frac{\sec^2(xy) - y}{x}$ (C) $\cos^2(xy)$

(D) $\frac{\cos^2(xy)}{x}$ (E) $\frac{\cos^2(xy) - y}{x}$

17.

If $f(x) = (2x+1)^4$, then the 4th derivative of $f(x)$ at $x = 0$ is

(A) 0 (B) 24 (C) 48 (D) 240 (E) 384

$f'(x) = 4(2x+1)^3 \cdot 2 = 8(2x+1)^3$

$f''(x) = 24(2x+1)^2 \cdot 2 = 48(2x+1)^2$

$f'''(x) = 96(2x+1) \cdot 2 = 192(2x+1)^1$

$f''''(x) = 384$

18.

If $y = \frac{3}{4+x^2}$, then $\frac{dy}{dx} =$ $\frac{(4+x^2)(0) - (3)(2x)}{(4+x^2)^2} = \frac{-6x}{(4+x^2)^2}$

(A) $\frac{-6x}{(4+x^2)^2}$ (B) $\frac{3x}{(4+x^2)^2}$ (C) $\frac{6x}{(4+x^2)^2}$ (D) $\frac{-3}{(4+x^2)^2}$ (E) $\frac{3}{2x}$

19.

The slope of the line tangent to the graph of $y = \ln\left(\frac{x}{2}\right)$ at $x = 4$ is

REWRITE: $y = \ln x - \ln 2$

(A) $\frac{1}{8}$ (B) $\frac{1}{4}$ (C) $\frac{1}{2}$ (D) 1 (E) 4

$y' = \frac{1}{x} - 0$

$y'(4) = \frac{1}{4}$

20.

$$2x + y + x\frac{dy}{dx} + 3y^2\frac{dy}{dx} = 0$$

If $x^2 + xy + y^3 = 0$, then, in terms of x and y, $\frac{dy}{dx} =$

$$\frac{dy}{dx}(x + 3y^2) = -2x - y$$

$$\frac{dy}{dx} = \frac{-2x-y}{x+3y^2} = \frac{-1(2x+y)}{x+3y^2}$$

(A) $-\frac{2x+y}{x+3y^2}$ (B) $-\frac{x+3y^2}{2x+y}$ (C) $\frac{-2x}{1+3y^2}$ (D) $\frac{-2x}{x+3y^2}$ (E) $-\frac{2x+y}{x+3y^2-1}$

21.

$\frac{d}{dx}\left(\frac{1}{x^3} - \frac{1}{x} + x^2\right)$ at $x = -1$ is

$$y = x^{-3} - x^{-1} + x^2$$

$$\frac{dy}{dx} = -3x^{-4} + x^{-2} + 2x$$

(A) -6 (B) -4 (C) 0 (D) 2 (E) 6

22.

$$\left.\frac{dy}{dx}\right|_{x=-1} = -3(-1)^{-4} + (-1)^{-2} + 2(-1) = -3 + 1 - 2 = -4$$

If $y = x^2e^x$, then $\frac{dy}{dx} =$ $e^x \cdot 2x + x^2 \cdot e^x = xe^x(\cancel{2x}2 + x)$

(A) $2xe^x$ (B) $x(x + 2e^x)$ (C) $xe^x(x+2)$

(D) $2x + e^x$ (E) $2x + e$

23.

If $y = \frac{\ln x}{x}$, then $\frac{dy}{dx} =$ $\frac{x \cdot \frac{1}{x} - \ln x}{x^2} = \frac{1 - \ln x}{x^2}$

(A) $\frac{1}{x}$ (B) $\frac{1}{x^2}$ (C) $\frac{\ln x - 1}{x^2}$ (D) $\frac{1 - \ln x}{x^2}$ (E) $\frac{1 + \ln x}{x^2}$

24.

$$1 + 2y + 2x\frac{dy}{dx} - 2y\frac{dy}{dx} = 0$$

If $x + 2xy - y^2 = 2$, then at the point $(1,1)$, $\frac{dy}{dx}$ is

$$\frac{dy}{dx} = \frac{-1-2y}{2x-2y} \rightarrow \left.\frac{dy}{dx}\right|_{(1,1)} = \frac{-1-2(1)}{2(1)-2(1)} = -\frac{3}{0}$$

(A) $\frac{3}{2}$ (B) $\frac{1}{2}$ (C) 0 (D) $-\frac{3}{2}$ (E) nonexistent

25.

An equation of the line tangent to the graph of $f(x) = x(1-2x)^3$ at the point $(1,-1)$ is

(A) $y = -7x + 6$ (B) $y = -6x + 5$ (C) $y = -2x + 1$

(D) $y = 2x - 3$ (E) $y = 7x - 8$

pt $\boxed{(1,-1)}$

slope $f'(x) = (1-2x)^3 + x \cdot 3(1-2x)^2 \cdot -2$

$f'(1) = (-1)^3 + 3(-1)^2 \cdot -2$

$= -1 - 6 = \boxed{-7}$

$y + 1 = -7(x-1)$

$y + 1 = -7x + 7$

$y = -7x + 6$