

$$45. f(x) = \arctan\left(\frac{x}{a}\right)$$

$$f'(x) = \frac{1/a}{1 + (x/a)^2}$$

$$47. g(x) = \frac{\arcsin(3x)}{x}$$

$$g'(x) = \frac{(x) \left(\frac{3}{\sqrt{1-(3x)^2}} \right) - (\arcsin 3x)(1)}{x^2}$$

$$= \frac{3x}{\sqrt{1-(3x)^2}} - \frac{\arcsin 3x}{x^2}$$

$$49. h(t) = \sin(\arccos t)$$

$$\frac{dh}{dt} = \cos(\arccos t) \cdot \frac{-1}{\sqrt{1-t^2}}$$

$$= -\frac{\cos(\arccos t)}{\sqrt{1-t^2}}$$

$$50. y = x \arccos x - \sqrt{1-x^2}$$

$$y' = (\arccos x)(1) + (x) \left(\frac{-1}{\sqrt{1-x^2}} \right) - \frac{1}{2} (1-x^2)^{-1/2} \cdot -2x$$

$$= \arccos x - \frac{x}{\sqrt{1-x^2}} + \frac{x}{\sqrt{1-x^2}} = \arccos x$$

$$53. y = \frac{1}{2} \left(\frac{1}{2} \ln \frac{x+1}{x-1} + \arctan x \right) \stackrel{\text{rewrite}}{=} \frac{1}{2} \left(\frac{1}{2} \ln x+1 - \frac{1}{2} \ln x-1 + \arctan x \right)$$

$$\frac{dy}{dx} = \frac{1}{2} \left(\frac{1}{2} \cdot \frac{1}{x+1} - \frac{1}{2} \cdot \frac{1}{x-1} + \frac{1}{1+x^2} \right)$$

$$= \frac{1}{2} \left(\frac{1}{2(x+1)} - \frac{1}{2(x-1)} + \frac{1}{1+x^2} \right)$$

$$55. y = x \arcsin x + \sqrt{1-x^2}$$

$$\frac{dy}{dx} = (\arcsin x)(1) + (x) \left(\frac{1}{\sqrt{1-x^2}} \right) + \frac{1}{2} (1-x^2)^{-1/2} \cdot -2x$$

$$= \arcsin x + \frac{x}{\sqrt{1-x^2}} - \frac{x}{\sqrt{1-x^2}}$$

$$= \arcsin x$$

$$57. y = 8 \sin^{-1}(x/4) - \frac{x\sqrt{16-x^2}}{2} \stackrel{\text{Rewrite}}{=} 8 \sin^{-1}(x/4) - \left(\frac{1}{2}x\right)(\sqrt{16-x^2})$$

$$y' = 8 \cdot \frac{1/4}{\sqrt{1-(x/4)^2}} - (\sqrt{16-x^2})(1/2) + (1/2x) \left(\frac{1}{2}(16-x^2)^{-1/2} \cdot -2x \right)$$

$$= \frac{2}{\sqrt{1-(x/4)^2}} - \frac{\sqrt{16-x^2}}{2} - \frac{x^2}{2\sqrt{16-x^2}}$$

61. point: $\left(\frac{1}{2}, \frac{\pi}{3}\right)$

slope: $y' = 2 \cdot \frac{1}{\sqrt{1-x^2}}$

$$y'(\frac{1}{2}) = 2 \cdot \frac{1}{\sqrt{1-(1/2)^2}} = \frac{2}{\sqrt{3/4}} = \boxed{\frac{4}{\sqrt{3}}}$$

$$y - \pi/3 = \frac{4}{\sqrt{3}}(x - \frac{1}{2})$$

62. point: $\left(-\frac{\sqrt{2}}{2}, \frac{3\pi}{8}\right)$

slope: $y' = \frac{1}{2} \cdot \frac{-1}{\sqrt{1-x^2}}$

$$y'(-\sqrt{2}/2) = \frac{1}{2} \cdot \frac{-1}{\sqrt{1-(\sqrt{2}/2)^2}} = \frac{1}{2} \cdot \frac{-1}{\sqrt{1/2}} = \frac{1}{2} \cdot \frac{-\sqrt{2}}{1} = \boxed{\frac{-\sqrt{2}}{2}}$$

$$y - \frac{3\pi}{8} = \frac{-\sqrt{2}}{2}(x + \sqrt{2}/2)$$

63. point: $(2, \pi/4)$

slope: $y' = \frac{1/2}{1+(x/2)^2}$

$$y'(2) = \frac{1/2}{1+(2/2)^2} = \frac{1/2}{1+1} = \boxed{\frac{1}{4}}$$

$$y - \pi/4 = 1/4 (x-2)$$

65. point: $(1, 2\pi)$

slope: $y' = (\arccos(x-1))' (4) + (4x) \left(\frac{-1}{\sqrt{1-(x-1)^2}} \right)$

$$y'(1) = (\arccos 0)' (4) + (4 \cdot 1) \left(\frac{-1}{\sqrt{1-(0)^2}} \right)$$

$$= (\pi/2)' (4) - 4 = \boxed{2\pi - 4}$$

$$y - 2\pi = (2\pi - 4)(x-1)$$

75. point: $(-\pi/4, 1)$

slope: $2x + (\arctan y)'(1) + (x) \left(\frac{1}{1+y^2} \cdot \frac{dy}{dx} \right) = \frac{dy}{dx}$

$$2x + \arctan y = \frac{dy}{dx} - \frac{x}{1+y^2} \frac{dy}{dx}$$

$$2x + \arctan y = \frac{dy}{dx} \left(1 - \frac{x}{1+y^2} \right)$$

$$\frac{dy}{dx} = \frac{2x + \arctan y}{1 - \frac{x}{1+y^2}}$$

$$\frac{dy}{dx} \Big|_{(-\pi/4, 1)} = \frac{2(-\pi/4) + \arctan(1)}{1 - \frac{-\pi/4}{1+(1)^2}} = \frac{-\pi/2 + \pi/4}{\frac{2+\pi/4}{2}} = \frac{\pi/4}{\frac{2+\pi/4}{2}} = \boxed{\frac{\pi}{4+\pi/2}}$$

$$y - 1 = \frac{\pi}{4+\pi/2} (x + \pi/4)$$

76. point: $(0,0)$

Slope:

$$\frac{(y)(1) + (x)\left(\frac{dy}{dx}\right)}{1 + (xy)^2} = \frac{1 + \frac{dy}{dx}}{\sqrt{1 - (x+y)^2}}$$

$$\left(\sqrt{1 - (x+y)^2}\right) \left(y + x \cdot \frac{dy}{dx}\right) = 1 + \frac{dy}{dx} + (xy)^2 + (xy)^2 \frac{dy}{dx}$$

$$\left(\sqrt{1 - (x+y)^2}\right) \left(y + x \cdot \frac{dy}{dx}\right) - \frac{dy}{dx} - (xy)^2 \frac{dy}{dx} = 1 + (xy)^2$$

$$y\sqrt{1 - (x+y)^2} + \sqrt{1 - (x+y)^2} \cdot x \cdot \frac{dy}{dx} - \frac{dy}{dx} - (xy)^2 \frac{dy}{dx} = 1 + (xy)^2$$

$$\sqrt{1 - (x+y)^2} \cdot x \cdot \frac{dy}{dx} - \frac{dy}{dx} - (xy)^2 \frac{dy}{dx} = -y\sqrt{1 - (x+y)^2} + 1 + (xy)^2$$

$$\frac{dy}{dx} \left(\sqrt{1 - (x+y)^2} \cdot x - 1 - (xy)^2 \right) = -y\sqrt{1 - (x+y)^2} + 1 + (xy)^2$$

$$\frac{dy}{dx} = \frac{-y\sqrt{1 - (x+y)^2} + 1 + (xy)^2}{\sqrt{1 - (x+y)^2} \cdot x - 1 - (xy)^2}$$

$$\left. \frac{dy}{dx} \right|_{(0,0)} = \frac{0 + 1 + 0}{0 - 1 - 0} = \frac{1}{-1} = \boxed{-1}$$

$$y - 0 = -1(x - 0)$$