Lab 7. Mattie's Mean Value Adventure

OBJECTIVE: To interpret the Mean Value Theorem graphically, numerically, and analytically

Mattie Mattic checked into the tollbooth at mile marker 0 on the Calculus Turnpike, and she checked out at the tollbooth at mile marker 150 exactly two hours later.

1. If the legal speed limit on this highway is e^4 miles/hour, prove Mattie should have been ticketed for speeding.

≈55 mph

(0, 0) (2, 150)

$\frac{150-0}{2-0}$ = 75 mph

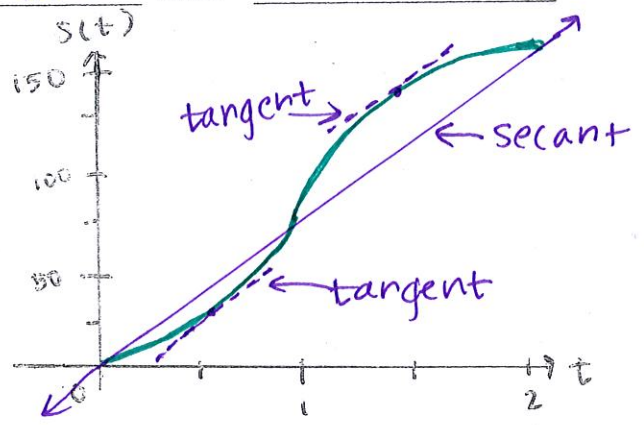
Mattie's average speed was 75 mph which means she went that speed at least once during the two hour drive.

2. Ian Der, the patrolman on duty, had more than just the Mean Value Theorem to support his legal action. He supplied Mattie with a model of her position function on the Calculus Turnpike. He told her that her position s at any time t, $0 \le t \le 2$, was $s(t) = -37.5t^3 + 112.5t^2$, where $s(t)$ is measured in miles and t in hours. Use your calculator to graph the position function for $0 \le t \le 2$ and $0 \le s \le 150$. To depict the Mean Value Theorem, calculator-draw a secant line connecting the endpoints of the position graph on the given time interval. Now, draw tangent lines that parallel the secant line, recording below the approximate times at which these tangent(s) occur.

Answers: At t = ≈ 0.5 sec and t = ≈1.5 sec

✓ _____

sketch s(t), tangent, and secant on given graph →



3. Build a table of values for Mattie's position function and corresponding velocity, based on the model from Problem 2:

t (time in hours)	s position (position in miles)	s' velocity (velocity in mph)
0	0	0
0.25	6.4453	49.219
0.50	23.438	84.375
0.75	47.461	105.47
1.00	75	112.5
1.25	102.54	105.47
1.50	126.56	84.375
1.75	143.55	49.219
2.00	150	0

← 75 mph

← 75 mph

Based on the table above, are your estimated times obtained in Problem 2 reasonable? Explain.

A closer estimate of the time she was going 75 mph is $t \approx 0.4$ and $t \approx 1.6$.

However, our estimates from problem 2 were not unreasonable

4. (a) You might not want to attempt to find graphically or numerically, using a table, the time when Mattie's speed was 75 mph (to the nearest thousandth of a second). Try another technique to find one of the times when her speed was 75 mph. Explain your approach and give the time answer accurate to 0.001 hr.

Answer: _____

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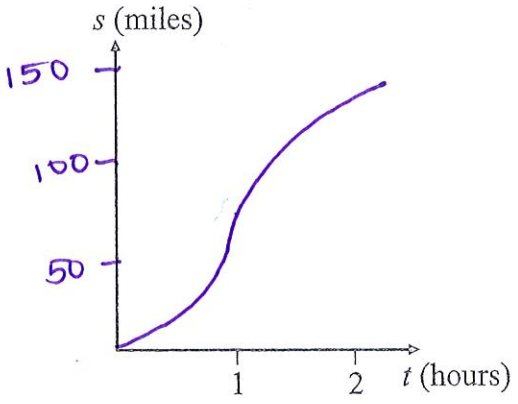
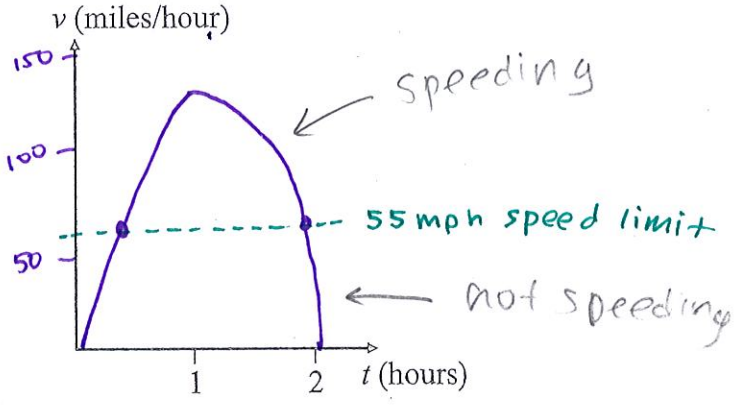
(b) Draw and label the position graph on the left axes, and the velocity graph and the speed limit graph on the right axes below, indicating a numerical scale on the vertical axes.





(c) Find the time interval over which Mattie Mattic could have been ticketed for speeding (recall that the speed limit is e^4 mph).

[.285, 1.715] hours } .0285 ≤ t ≤ 1.715 hours

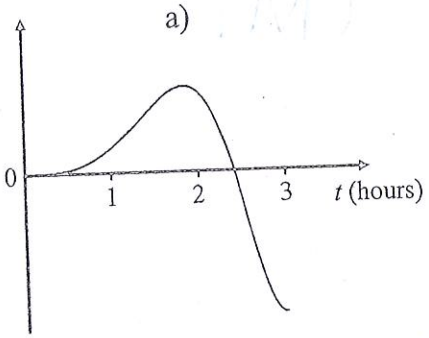
(d) When is Mattie's speed the greatest? t=1 hour Justify your answer.

using graphing calculator, find maximum on velocity graph [V(1) = 112.499 mph] speed at t=1 hour

Technology Stretch

The following graphs are the position, velocity, and acceleration for Mattie, riding her bicycle on a different trip, over a time interval from $t = 0$ to $t = 3$ hours. The graphs, shown below, represent Mattie's position, velocity, and acceleration, not necessarily in that order:

a)



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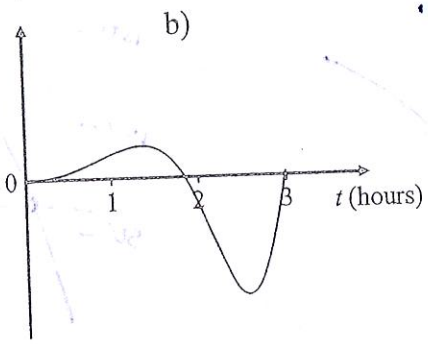
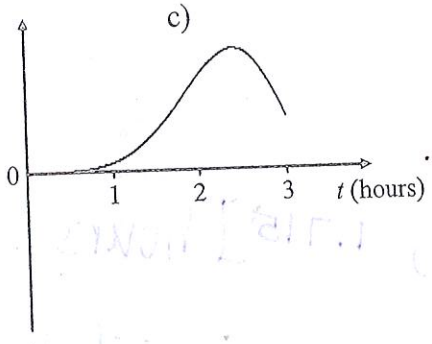
b)



a

c)



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Which graph is which?

Position: c; velocity: a; acceleration: b.