____________ 1. A function f is continuous for all x and has a local minimum at (2, -5). Which statement **must** be true?

a. $f'(2) = 0$

b. $f'(x)$ exists at $x = 2$

c. the graph is concave up at $x = 2$

d. $f'(x)$ is positive if $x < 2$ and $f'(x)$ is negative if $x > 2$

e. $f'(x)$ is positive if $x > 2$ and $f'(x)$ is negative if $x < 2$

2. Find the absolute maximum and absolute minimum of the function $f(x) = x^3 - x^2 - x + 2$ on the interval [-10, 2]. Verify using calculus.

EVT

$f'(x) = 3x^2 - 2x - 1 = 0$
$= (3x+1)(x-1) = 0$
$x = -1/3, 1$

We have 2 maxes: $x = -1/3$ & $x = 2$
$f(-1/3) = 1.5$ $f(2) = 4$

abs max at $x = 2$
abs min at $x = -10$

SKIP ~~3. Let $f(x) = ax + \frac{b}{x}$ where a and b are positive constants. Find in terms of a and b the x-coordinate of all local maximum and minimum. Justify.~~

$f'(x) = a - \frac{b}{x^2} = 0$
$a = \frac{b}{x^2}$
$x = -\sqrt{b/a}, \sqrt{b/a}$ critical pts

min: $x = -\sqrt{b/a}$
max: $x = \sqrt{b/a}$

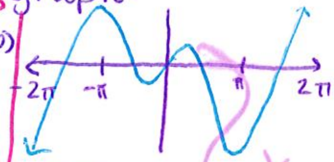
4. Given $f' = x\cos(x)$; $-2\pi \le x \le 2\pi$. Justify answers. Calculator allowed.

where $f' > 0$ → i. On what intervals is f increasing? $(-4.7, -\pi/2)$ + $(0, \pi/2)$ + $(4.7, \infty)$

f' is increasing ii. On what intervals is the graph of f concave up? $(-\infty, -3.4)$ $(-.86, .86)$ $(4.7, \infty)$

& where $f' = 0$ or DNE & changes sign → iii. Where does f have pints of inflection?
$x = \pm 3.426$ & ± 0.86

graph:



5. For what value of k will $x + \frac{k}{x}$ have a relative maximum at x = -2?

(a) -4 (b) -2 (c) 2 (d) 4 (e) None of these

$f(x) = x + \frac{k}{x}$
$f'(x) = 1 - \frac{k}{x^2} = 0$
$f'(-2) = 1 - \frac{k}{(-2)^2} = 0$
$k = 4$

6. At x = 0, which of the following is true of the function f defined by $f(x) = x^2 + e^{-2x}$?

(a) f is increasing (b) f is decreasing (c) f is discontinuous
(d) f has a relative minimum (e) f has a relative maximum

$f'(x) = 2x - 2e^{-2x}$
$f'(0) = 2(0) - 2e^{-2(0)}$
$= 0 - 2$
$= -2$

$f'(0) < 0$ so f is decreasing

7. If $f(x) = \frac{x^3}{3} - 4x^2 + 12x - 5$ and the domain is the set of all x such that $0 \le x \le 9$, then the absolute maximum value of the function f occurs when x is

(a) 0 (b) 2 (c) 4 (d) 6 (e) 9

$f'(x) = x^2 - 8x + 12 = 0$
$(x-6)(x-2) = 0$
$x = 2, 6$

there is a closed interval so $x = 0$ & $x = 9$ MUST be a min or max

We have two maxes: $x = 2$, $x = 9$
$f(2) = 5.7$
$f(9) = 22$ } abs. max at $x = 9$

8. Given that $f(x) = x^3 + ax^2 + x$ has a critical number at $x = 1$, find a.

$f'(x) = 3x^2 + 2ax + 1 = 0$

$f'(1) = 3(1)^2 + 2a(1) + 1 = 0$

$3 + 2a + 1 = 0$

$2a = -4$

$a = -2$

9. On what interval is the function $f(x) = \dfrac{x}{x^2 + 1}$ increasing?

a. (-1, 1) b. (-1, 2) c. $(-\infty, -1) \cup (1, \infty)$ d. $(-\infty, -1) \cup (2, \infty)$

$f'(x) = \dfrac{(x^2+1)(1) - (x)(2x)}{(x^2+1)^2} = 0$

$\dfrac{x^2 + 1 - 2x^2}{(x^2+1)^2} = 0$

$\dfrac{-x^2+1}{(x^2+1)^2} = 0$

$-x^2 + 1 = 0$

$x^2 = 1$

$x = 1, -1$

increasing

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d. $f'(x)$ is positive if $x < 2$ and $f'(x)$ is negative if $x > 2$

e. $f'(x)$ is positive if $x > 2$ and $f'(x)$ is negative if $x < 2$

repeat #1

oops!