OPTIMIZATION

1. Let S be the set of all rectangles with area equal to 100. What are the dimensions of the rectangle in S with the least perimeter?

2. Kayla is planning to make an open rectangular box from a 40 by 50 cm piece of cardboard by cutting congruent squares from corners and folding up the sides.

a. What are the dimensions of the box of largest volume that she can make this way?

b. What is its volume?

3. Help Avalon find the dimensions of the rectangle with the largest area that can be inscribed in the region bounded by the curve $y = \sqrt{6-x}$ in the first quadrant.

4. Find the point on the curve $y = \sqrt{x}$ that is closest to the point (3,0).

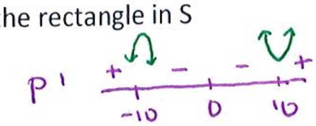
5. A box with a square base and open top must have a volume of 32,000 cm^3. Find the dimensions of the box that minimize the amount of material used.

6. If 1,2000 cm^2 of material is available to make a box with a square base and an open top, find the largest possible volume of the box.

OPTIMIZATION

1. Let S be the set of all rectangles with area equal to 100. What are the dimensions of the rectangle in S with the least perimeter?

MINIMIZE



$A = xy = 100$

$y = \frac{100}{x}$

$P = 2x + 2y$

$= 2x + 2\left(\frac{100}{x}\right)$

$= 2x + 200x^{-1}$

$P' = 2 - 200x^{-2} = 0 / DNE$

$2 = \frac{200}{x^2}$

$2x^2 = 200$

$x^2 = 100$

$x = \pm 10, 0$

$\boxed{\text{Dimensions: } 10 \times 10}$

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MAX

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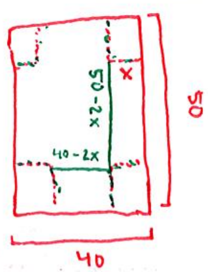
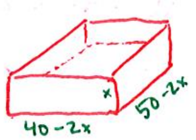
b. What is its volume?





$V = lwh$

$= (40-2x)(50-2x)(x)$

$= (40-2x)(50x-2x^2)$

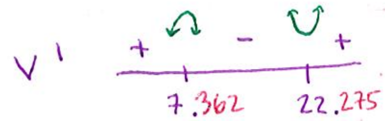
$= 2000x - 80x^2 - 100x^2 + 4x^3$

$= 4x^3 - 180x^2 + 2000x$

$V' = \underset{a}{12x^2} \underset{b}{- 360x} + \underset{c}{2000} = 0/DNE$

$x = \frac{360 \pm \sqrt{(-360)^2 - 4(12)(2000)}}{2(12)}$

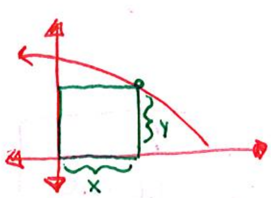
$x = \frac{360 \pm \sqrt{33600}}{24} = 7.362 + 22.638$



$\boxed{\begin{array}{l} a.\ 7.362 \times 25.275 \times 35.275 \text{ centimeters} \\ b.\ 6564.226\ cm^3 \end{array}}$

3. Help Avalon find the dimensions of the rectangle with the largest area that can be inscribed in the region bounded by the curve $y = \sqrt{6-x}$ in the first quadrant.

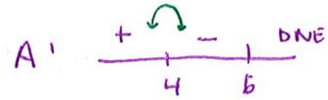
MAX



$A = xy$

$= x(\sqrt{6-x})$

$= x(6-x)^{1/2}$



$A' = (6-x)^{1/2} \cdot 1 + x \cdot \frac{1}{2}(6-x)^{-1/2} \cdot -1 = 0/DNE$

$\sqrt{6-x} - \frac{x}{2\sqrt{6-x}} = 0/DNE$

$\sqrt{6-x} = \frac{x}{2\sqrt{6-x}}$

$2(6-x) = x$

$12 - 2x = x$

$12 - 3x = 0$

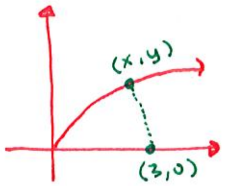
$3(4-x) = 0$

$x = 4, 6$

$\boxed{\text{Dimensions: } 4 \times \sqrt{2}}$

4. Find the point on the curve $y=\sqrt{x}$ that is $\boxed{\text{closest}}$ to the point (3,0).

MIN



points: $(\overset{x_1}{3},\overset{y_1}{0})$ $(\overset{x_2}{x},\overset{y_2}{\sqrt{x}})$

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$=\sqrt{(x-3)^2+(\sqrt{x}-0)^2}$$
$$=\sqrt{x^2-6x+9+x}$$
$$=\sqrt{x^2-5x+9}$$

$$d' = \frac{1}{2}(x^2-5x+9)^{-1/2}\cdot(2x-5)=0/DNE$$
$$\frac{2x-5}{2\sqrt{x^2-5x+9}}=0\ /DNE$$
$$x=2.5$$

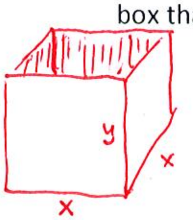
discriminant is neg. value so no roots exist from denom.

d': $-$ ∪ $+$ at 2.5

$\boxed{\text{closest pt: } (2.5, \sqrt{2.5})}$

5. A box with a square base and open top must have a volume of 32,000 cm^3. Find the dimensions of the box that $\boxed{\text{minimize}}$ the amount of material used.

MIN



$$V=lwh=32{,}000$$
$$x^2y=32{,}000$$
$$y=\frac{32{,}000}{x^2}$$

$$SA=4xy+x^2$$
$$=4x\left(\frac{32000}{x^2}\right)+x^2$$
$$=\frac{128{,}000}{x}+x^2$$
$$=128{,}000x^{-1}+x^2$$

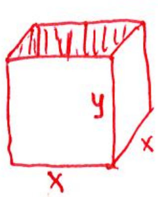
SA': $-$ at 0, $-$ ∪ $+$ at 40

$$SA'=-128{,}000x^{-2}+2x=0/DNE$$
$$\frac{-128{,}000}{x^2}+2x=0/DNE$$
$$\frac{-128{,}000}{x^2}=-2x$$
$$-2x^3=-128{,}000$$
$$x^3=64{,}000$$
$$x=\sqrt[3]{64{,}000}=40{,}0$$

$\boxed{\text{dimensions: } 40\times40\times20 \text{ cm}}$

6. If 1,200 cm^2 of material is available to make a box with a square base and an open top, find the $\boxed{\text{largest}}$ possible volume of the box.

MAX



$$SA=4xy+x^2=1200$$
$$4xy=1200-x^2$$
$$y=\frac{1200-x^2}{4x}$$
$$y=\frac{300}{x}-\frac{x}{4}$$
$$y=300x^{-1}-\frac{1}{4}x$$

$$V=lwh$$
$$=x^2y$$
$$=x^2\left(300x^{-1}-\frac{1}{4}x\right)$$
$$=300x-\frac{1}{4}x^3$$

$$V'=300-\frac{3}{4}x^2=0\ /DNE$$
$$300=\frac{3}{4}x^2$$
$$1200=3x^2$$
$$400=x^2$$
$$x=\pm20$$

V': $-$ ∪ $+$ ∩ $-$ at -20, 20

$\boxed{\text{Volume: } 20\times20\times10 = 4{,}000 \text{ cm}^3}$