Calculus Name________________________________

Finding Increasing and Decreasing Intervals

Date________________ Period____

For each problem, find the open intervals where the function is increasing and decreasing.

2) $f(x) = -\dfrac{2x}{x-1}$

3) $y = -\dfrac{3x}{x+2}$

4) $y = -x^2$

5) $f(x) = \csc(x);\ [-\pi, \pi]$

6) $f(x) = 2x^2 - 4x + 4$

7) $f(x) = -x^5 + 3x^3$

8) $f(x) = -(6x+6)^{\frac{1}{2}}$

9) $y = -x^5 + 3x^3 + 1$

10) $y = x^4 + 4x^3 + 2x^2 - 4x - 5$

11) $y = -x^5 + 2x^3 + 3$

12) $f(x) = -(3x-9)^{\frac{1}{3}}$

13) $y = x^2 + 4x - 2$

~~$10)\ y' = 4x^3 + 12x^2 + 4x - 4 = 0$~~

2. $f'(x) = \dfrac{(x-1)(-2) - (-2x)(1)}{(x-1)^2} = 0/DNE$

$\dfrac{\cancel{-2x} + 2 + \cancel{2x}}{(x-1)^2} = 0/DNE$

$x = 1$

$f'(x)$: $+$ | $+$; 1

incr: $(-\infty, \infty)$
decr: none

3. $y' = \dfrac{(x+2)(-3) - (-3x)(1)}{(x+2)^2} = 0/DNE$

$\dfrac{\cancel{-3x} - 6 + \cancel{3x}}{(x+2)^2} = 0/DNE$

$x = -2$

y': $-$ | $-$; -2

dec: $(-\infty, \infty)$
inc: none

4. $y' = -2x = 0/DNE$

$x = 0$

y': $+$ | $-$; 0

inc: $(-\infty, 0)$
dec: $(0, \infty)$

5. $f'(x) = -\csc x \cot x = 0/DNE$

$\dfrac{-1}{\sin x} \cdot \dfrac{\cos x}{\sin x} = 0/DNE$

$x = -\pi, -\pi/2, 0, \pi/2, \pi$

f': $+$ | $-$ | $-$ | $+$; $-\pi$, $-\frac{\pi}{2}$, 0, $\frac{\pi}{2}$, π

inc: $(-\pi, -\pi/2)$ $(\pi/2, \pi)$

dec: $(-\pi/2, \pi/2)$

6. $f' = 4x - 4 = 0/DNE$

$4(x-1) = 0/DNE$

$x = 1$

f': $-$ | $+$; 1

inc: $(1, \infty)$
dec: $(-\infty, 1)$

7. $f' = -5x^4 + 9x^2 = 0/DNE$

$x^2(-5x^2 + 9) = 0/DNE$

$x = 0, \pm\sqrt{9/5}$

$= 0, \pm \frac{3}{\sqrt{5}}$

f': $-$ | $+$ | $+$ | $-$; $\frac{-3}{\sqrt{5}}$, 0, $\frac{3}{\sqrt{5}}$

81/81 =

incr: $(-\frac{3}{\sqrt{5}}, \frac{3}{\sqrt{5}})$
decr: $(-\infty, -\frac{3}{\sqrt{5}})$ $(\frac{3}{\sqrt{5}}, \infty)$

8. $f'(x) = -\frac{1}{2}(6x+6)^{-1/2} \cdot 6 = 0$

$\dfrac{-3}{\sqrt{6x+6}} = 0/DNE$

$x = -1$

f': DNE | $-$; -1

inc: none
dec: $(-1, \infty)$

9. $y' = -5x^4 + 9x^2 = 0/DNE$

same as #9

$x^2(-5x^2 + 9) = 0/DNE$

$x = 0, \pm\sqrt{9/5}$

f': $-$ | $+$ | $+$ | $-$; $\frac{-3}{\sqrt{5}}$, 0, $\frac{3}{5}$

~~$10)\ y' = 4x^3 + 12$~~

inc: $(-\frac{3}{\sqrt{5}}, \frac{3}{\sqrt{5}})$
dec: $(-\infty, -\frac{3}{\sqrt{5}})$
$(\frac{3}{\sqrt{5}}, \infty)$

challenge problem

10. $y' = 4x^3 + 12x^2 + 4x - 4 = 0/DNE$

$4(x^3 + 3x^2 + x - 1) = 0$

$x = -1$

now we know $(x+1)$ is a root of y so divide:

$$\cancel{x+1 \overline{)x^4 + 4x^3 + 2x^2 - 4x - 5}}$$

$$
\begin{array}{r}
x^2 + 2x - 1 \\
x+1 \overline{)x^3 + 3x^2 + x - 1} \\
\underline{x^3 + x^2} \\
2x^2 + x \\
\underline{2x^2 + 2x} \\
-x - 1 \\
\underline{-x - 1} \\
0
\end{array}
$$

$y' = 4\underbrace{(x+1)}_{x=-1}\underbrace{(x^2+2x-1)}_{\text{quadratic formula}}$

$x = \dfrac{-2 \pm \sqrt{4 - 4(1)(-1)}}{2(1)}$

$= \dfrac{-2 \pm \sqrt{8}}{2} = \dfrac{-2 \pm 2\sqrt{2}}{2}$

dec inc dec inc

$- \quad + \quad - \quad +$

$\frac{-2-2\sqrt{2}}{2} \quad -1 \quad \frac{-2+2\sqrt{2}}{2}$

11. $y' = -5x^4 + 6x^2 = 0/DNE$

$x^2(-5x^2 + 6) = 0$

$x = 0, \pm\sqrt{6/5}$

y': $- \quad + \quad + \quad -$

$-\sqrt{\frac{6}{5}} \quad 0 \quad \sqrt{\frac{6}{5}}$

inc: $(-\sqrt{6/5}, \sqrt{6/5})$

dec: $(-\infty, -\sqrt{\frac{6}{5}})$ $(\sqrt{\frac{6}{5}}, \infty)$

12. $f'(x) = -\frac{1}{3}(3x-9)^{-2/3} \cdot 3 = 0/DNE$

$\dfrac{-1}{(3x-9)^{2/3}} = 0/DNE$

$x = 3$

f': $- \quad -$

3

inc: none

dec: $(-\infty, \infty)$

13. $f'(x) = 2x + 4 = 0/DNE$

$x = -2$

f': $- \quad +$

-2

inc: $(-2, \infty)$

dec: $(-\infty, -2)$