minimize.

1. Let S be the set of all rectangles with area = 100. What are the dimensions of the rectangle in S with the least perimeter?

Area: $x \cdot y = 100$

so $y = \frac{100}{x}$

$P = 2x + 2y$
$= 2x + 2\left(\frac{100}{x}\right)$
$= 2x + \frac{200}{x}$

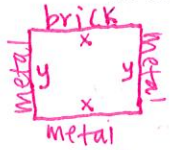
$P' = 2 - \frac{200}{x^2} = 0$

$\frac{200}{x^2} = 2$

$x = 0, \pm 10$

At $x=10$, P has a minimum. So $x=10$ + $y=10$

2. A landscape architect, Lili, wishes to enclose a rectangular garden on one side by a brick wall costing $30/feet and the other 3 sides by a metal fence costing $10/ft. If the area of the garden is 1000 ft², find the dimensions of the garden that minimize the cost. ← minimize!



$A = x \cdot y = 1000$

$y = \frac{1000}{x}$

$P = x + x + y + y$
$= x + x + \left(\frac{1000}{x}\right) + \left(\frac{1000}{x}\right)$
$= 30x + 10x + 10\left(\frac{1000}{x}\right) + 10\left(\frac{1000}{x}\right)$
$= 40x + \frac{20000}{x}$

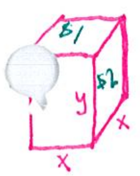
$P' = 40 - \frac{20000}{x^2} = 0$

$\frac{20000}{x^2} = 40$

$x = \pm\sqrt{500}, 0$

At $x=500$, P has a minimum. so $x = \sqrt{500}$ + $y = \frac{1000}{\sqrt{500}}$

3. A box is constructed out of 2 different types of metal. The metal on top and bottom, which are both square, costs $1.00/ft² and the metal for the sides cost $2.00/ft². Find the dimensions that minimize cost if the box has a volume of 20 ft³. minimize!



$V = x^2 y = 20$

$y = \frac{20}{x^2}$

$SA = 4xy + 2x^2$
$= 4x\left(\frac{20}{x^2}\right) + 2x^2$
$= \frac{80}{x} + 2x^2$
$= 2\left(\frac{80}{x}\right) + 1\left(2x^2\right)$
$= \frac{160}{x} + 2x^2$

$SA' = -\frac{160}{x^2} + 4x = 0$

$4x = \frac{160}{x^2}$

$4x^3 = 160$

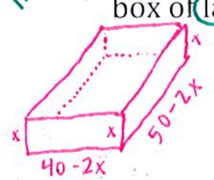
$x^3 = 40$

$x = 0, \sqrt[3]{40}$

At $x=10$, SA has a min. So $x = \sqrt[3]{40}$ + $y = \frac{20}{(\sqrt[3]{40})^2}$

maximize!

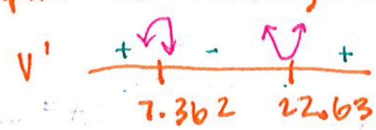
4. Kayla is planning to make an open rectangular box from a 40 by 50 cm piece of cardboard by cutting congruent squares from the corners and folding up the sides. What are the dimensions of the box of largest volume that she can make this way? What is its volume?



$V = l \cdot w \cdot h$
$= (40-2x)(50-2x)(x)$
$= (40-2x)(50x-2x^2)$

$V' = (50x-2x^2)(-2) + (40-2x)(50-4x) = 0$

graph: $x = 7.362, 22.637$

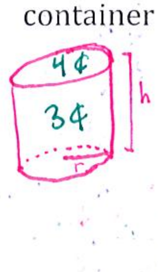


$l = 40-2x$
$w = 50-2x$
$h = x$

Volume has a MAX at $x = 7.362$, so $l = 35.275$ cm, $w = 25.275$ cm, & $h = 7.362$ cm. $V = 6564.225$ cm³

minimize!

5. Find the minimum cost to construct a cylindrical container if the material for the top and bottom costs 4 cents per square inch and the material for the sides costs 3 cents per square inch. The container is to have a volume of 100 cubic inches.



$V = \pi r^2 \cdot h = 100$

$h = \frac{100}{\pi r^2}$

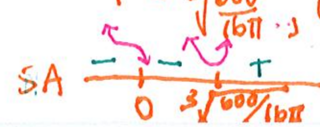
$SA = 2\pi r^2 + 2\pi r h$
$= 2\pi r^2 + 2\pi r\left(\frac{100}{\pi r^2}\right)$
$= 2\pi r^2 + \frac{200}{r}$
$= 4\left(2\pi r^2\right) + 3\left(\frac{200}{r}\right)$
$= 8\pi r^2 + \frac{600}{r}$

$SA' = 16\pi r - \frac{600}{r^2} = 0$

$16\pi r^3 = 600$

$r^3 = \frac{600}{16\pi}$

$r = \sqrt[3]{\frac{600}{16\pi}}, 0$

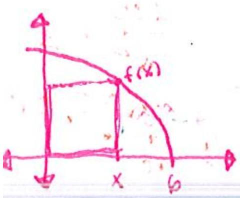


There is a min at $x = \sqrt[3]{\frac{600}{16\pi}}$ so min cost is 393.84¢ which is $3.93

Picture is crucial!

maximize!

6. Help Avalon find the dimensions of the rectangle with the largest area that can be inscribed in the region bounded by the curve $y=\sqrt{6-x}$ in the first quadrant.



Area $= x \cdot y$

$= x \cdot f(x)$

$= x\sqrt{6-x}$

$A' = \sqrt{6-x}\cdot 1 + x \cdot \frac{1}{2}(6-x)^{-1/2}\cdot -1 = 0$

$= \sqrt{6-x} - \frac{x}{2\sqrt{6-x}} = 0$

$2(6-x) = x$

$12-2x = x$

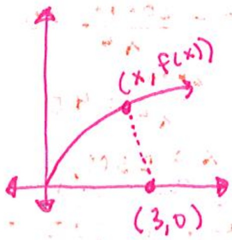
$12 = 3x \rightarrow x = 4, 6$

A': $+$ $-$ at 4, DNE at 6

A has a max at $x=4$, so $x=4$ and $y=\sqrt{2}$

7. Find the point on the curve $y=\sqrt{x}$ that is closest to the point (3, 0).

minimize!



pt $(3,0)$ + $(x, \sqrt{x})$

$d = \sqrt{(3-x)^2 + (0-\sqrt{x})^2}$

$= \sqrt{9-6x+x^2+x}$

$= \sqrt{x^2-5x+9}$

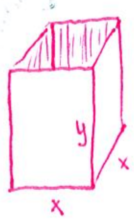
$d' = \frac{1}{2}(x^2-5x+9)^{-1/2}\cdot \underbrace{(2x-5)}_{x=5/2} = 0$

d': $-$ $+$ at $5/2$

D has a min at $x=5/2$, so the closest pt is $(5/2, \sqrt{5/2})$

minimize!

8. A box with a square base and open top must have a volume of 32,000 cm^3. Find the dimensions of the box that minimize the amount of material used.



$V = x^2y = 32000$

$y = \frac{32000}{x^2}$

$SA = x^2 + 4xy$

$= x^2 + 4x\left(\frac{32000}{x^2}\right)$

$= x^2 + \frac{128000}{x}$

$SA' = 2x - \frac{128000}{x^2} = 0$

$2x = \frac{128000}{x^2}$

$2x^3 = 128000$

$x^3 = 64000$

$x = \sqrt[3]{6400}, 0$

SA': $-$ $-$ $+$ at 0, $\sqrt[3]{6400}$

SA has a min at $x = \sqrt[3]{6400} = 18.566$. So length + width $= 18.566$ + height $= 92.832$

$(\sqrt{2r^2})$

$= 2r^2$

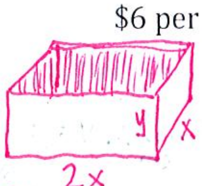
$y^2 = r^2 + r^2$

$y = \sqrt{2r^2}$

$A_0 = \pi r^2$

10. A rectangular storage container with an open top is to have a volume of 10 m^3. The length of its base is twice the width. Material for the base costs \$10 per square meter. Material for the sides cost \$6 per square meter. Find the cost of materials for the cheapest such container.

minimize!



$V = 2x^2y = 10$

$y = \frac{10}{2x^2} = \frac{5}{x^2}$

$SA = 2x^2 + 6xy$

$= 2x^2 + 6x(5/x^2)$

$= 10(2x^2) + 6\left(\frac{30}{x}\right)$

$= 20x^2 + \frac{180}{x}$

$SA' = 40x - \frac{180}{x^2} = 0$

$40x = \frac{180}{x^2}$

$40x^3 = 180$

$x^3 = 9/2$

$x = \sqrt[3]{9/2}, 0$

SA': $-$ $-$ $+$ at 0, $\sqrt[3]{9/2}$

SA has a min at $x = \sqrt[3]{4.5}$

Cost is \$6903.88

Maximize

11. If 1200 cm^2 of material is available to make a box with a square base and an open top, find the largest possible volume of the box.

$SA = 4xy + x^2 = 1200$

$y = \frac{1200 - x^2}{4x}$

$= \frac{300}{x} - \frac{1}{4}x$

$V = x^2 \cdot y$

$= x^2\left(\frac{300}{x} - \frac{1}{4}x\right)$

$= 300x - \frac{1}{4}x^3$

$V' = 300 - \frac{3}{4}x^2 = 0$

$\frac{3}{4}x^2 = 300$

$x^2 = 400$

$x = \pm 20$

V': $-$ $+$ $-$ at -20, 20

V has a max at $x = 20$. Largest volume is $4000\,cm^3$.