U5H2 OPTIMIZATION

goal OPTIMIZE MAX

1. Find two numbers whose sum is 23 and whose product is a maximum.

Sum: $x + y = 23$

$y = 23 - x$

PRODUCT: xy

$P = x(23 - x)$

$= 23x - x^2$

$P' = 23 - 2x = 0$

$-2x = -23$

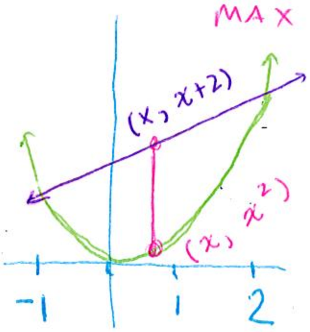
$x = 11.5$

P': $+$ | $-$ at 11.5

$\boxed{\text{Answer: } 11.5 \text{ and } 11.5}$

2. What is maximum vertical distance between the line $y = x + 2$ and the parabola $y = x^2$ for $-1 \le x \le 2$?

MAX OPTIMIZE



$d = (x+2) - (x^2)$

$= x + 2 - x^2$

$d' = 1 - 2x = 0/DNE$

$x = 1/2$

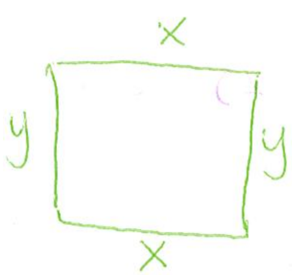
$+$ | $-$ at $\frac{1}{2}$

plug $x = 1/2$ into "d" to find distance

$\boxed{\text{Answer: } 2.25}$

goal OPTIMIZE MAX

3. Find the dimensions of a rectangle with perimeter 100m whose area is as large as possible.



P: $2x + 2y = 100$

$y = \frac{100 - 2x}{2}$

$y = 50 - x$

Area $= xy$

$A = x(50 - x)$

$= 50x - x^2$

$A' = 50 - 2x = 0$

$50 = 2x$

$x = 25$

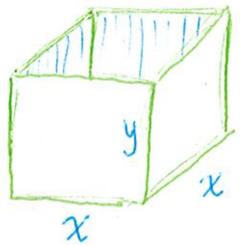
A': $+$ | $-$ at 25

$\boxed{\text{Answer: } 25_m \text{ by } 25_m}$

MAX

4. 1200 cm^2 of material is available to make a box with a square base and an open top. Find largest possible volume of the box.

OPTIMIZE



$$SA = x^2 + 4xy = 1200$$

$$y = \frac{1200 - x^2}{4x}$$

$$y = \frac{300}{x} - \frac{x}{4}$$

$$V = x^2 y$$

$$= x^2\left(\frac{300}{x} - \frac{x}{4}\right)$$

$$= 300x - \frac{x^3}{4}$$

$$V' = 300 - \frac{3}{4}x^2 = 0$$

$$400 = x^2$$

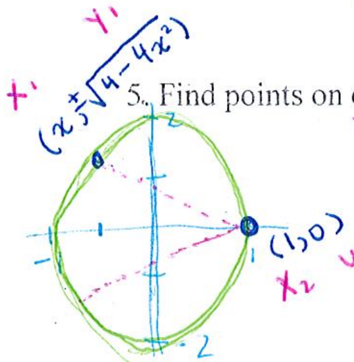
$$x = \pm 20$$

V': $-$ | -20 | $+$ | 20 | $-$

$$\boxed{\text{Answer: } V = 4000 \text{ cm}^3}$$

OPTIMIZE + MAX

5. Find points on ellipse $4x^2 + y^2 = 4$ that are farthest away from point (1, 0).



$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(x-1)^2 + \left(\pm\sqrt{4-4x^2}\right)^2}$$

$$= \sqrt{x^2 - 2x + 1 + 4 - 4x^2}$$

$$= \sqrt{-3x^2 - 2x + 5}$$

$$d' = \frac{1}{2}(-3x^2 - 2x + 5)^{-1/2} \cdot (-6x - 2) = 0$$

$$\frac{-6x - 2}{2\sqrt{-3x^2 - 2x + 1}} = 0$$

$$\cancel{2\sqrt{-3x^2 - 2x + 1} = 0}$$

$$\cancel{-3x^2 - 2x + 1 = 0}$$

$$\cancel{(3x - 1)(-x - 1)}$$

$$\cancel{x = 1,}$$

from discriminant, there are no x-values from denominator

$$-6x - 2 = 0$$

$$2 = -6x$$

$$x = -1/3$$

d': $+$ | $-1/3$ | $-$

$$\boxed{\text{Answer: } \left(-\frac{1}{3}, \pm\sqrt{\frac{32}{9}}\right)}$$