Analyzing Functions Using Derivatives Name *key*

Find the intercals on which f is increasing or decreasing. Find the local (relative) maximum and minimum values of f. Find the intervals of concavity and the inflection points.

1. $f(x) = 2x^3 + 3x^2 - 36x$

$f'(x) = 6x^2 + 6x - 36 = 0/DNE$

$6(x^2 + x - 6) = 0$

$6(x+3)(x-2) = 0$

$\boxed{x = -3, 2}$ CPs

f': + (−3) − (2) +

$f''(x) = 12x + 6 = 0/DNE$

$6(2x+1) = 0$

$\boxed{x = -1/2}$ POSSIBLE IP

f'': − $\left(-\frac{1}{2}\right)$ +

critical pt(s):	$x = -3, 2$
int. of inc:	$(-\infty, -3) \cup (2, \infty)$
int. of dec:	$(-3, 2)$
extrema:	$x = -3$: relative max $x = 2$: relative min
infl. pt(s):	$x = -1/2$
int. con. up:	$(-1/2, \infty)$
int. con. down:	$(-\infty, 1/2)$

2. $f(x) = 4x^3 + 3x^2 - 6x + 1$

$f'(x) = 12x^2 + 6x - 6 = 0/DNE$

$6(2x^2 + x - 1) = 0$

$6(2x-1)(x+1) = 0$

$\boxed{x = 1/2, -1}$ CPs

f': + (−1) − $\left(\frac{1}{2}\right)$ +

$f''(x) = 24x + 6 = 0/DNE$

$6(4x+1) = 0$

$\boxed{x = -1/4}$ POSSIBLE IP

f'': − $\left(-\frac{1}{4}\right)$ +

critical pt(s):	$x = -1, 1/2$
int. of inc:	$(-\infty, -1) \cup (1/2, \infty)$
int. of dec:	$(-1, 1/2)$
extrema:	$x = -1$: relative max $x = 1/2$: relative min
infl. pt(s):	$x = -1/4$
int. con. up:	$(-1/4, \infty)$
int. con. down:	$(-\infty, -1/4)$

3. $f(x) = \dfrac{x}{x^2+1}$

$f'(x) = \dfrac{(x^2+1)(1) - (x)(2x)}{(x^2+1)^2} = 0/DNE$

$\dfrac{x^2 + 1 - 2x^2}{(x^2+1)^2} = 0$

$\dfrac{-x^2+1}{\cancel{(x^2+1)^2}} = 0$

in this case, no values will come from denominator

$\boxed{x = \pm 1}$ CPs

$f'(x)$: − (−1) + (1) −

$f''(x) = \dfrac{(x^2+1)^2(-2x) - (-x^2+1)(2(x^2+1)\cdot 2x)}{(x^2+1)^4} = 0/DNE$

$\dfrac{\cancel{(x^2+1)}\left[(x^2+1)(-2x) - (x^2+1)(2)(2x)\right]}{(x^2+1)^{3}} = 0$

$\dfrac{-2x^3 - 2x - 4x + 4x^3}{(x^2+1)^3} = 0$

$\dfrac{2x(x^2-3)}{\cancel{(x^2+1)^3}} = 0$

in this case, no values will come from denominator

$\boxed{x = 0, \pm\sqrt{3}}$ POSSIBLE IPs

f'': − $(-\sqrt{3})$ + (0) − $(\sqrt{3})$ +

critical pt(s):	$x = \pm 1$
int. of inc:	$(-1, 1)$
int. of dec:	$(-\infty, -1) \cup (1, \infty)$
extrema:	$x = -1$: relative min $x = 1$: relative max
infl. pt(s):	$x = 0, \pm\sqrt{3}$
int. con. up:	$(-\sqrt{3}, 0) \cup (\sqrt{3}, \infty)$
int. con. down:	$(-\infty, -\sqrt{3}) \cup (\sqrt{3}, \infty)$

4. $f(x) = \cos^2(x) - 2\sin(x)$ $[-\pi, \pi]$

$f'(x) = 2\cos x \cdot -\sin x - 2\cos x = 0/DNE$

$\underbrace{-2\cos x}\,(\underbrace{\sin x + 1}) = 0$

$-2\cos x = 0$

$\cos x = 0$

$\boxed{x = \pm \pi/2}$ CPs

$\sin x + 1 = 0$

$\sin x = -1$

$x = \cancel{3\pi/2} = -\pi/2$

not in interval

f': $-\pi$ + $-\frac{\pi}{2}$ − $\frac{\pi}{2}$ + π

$f(-\pi) = 1$

$f(\pi/2) = -2$

$f(-\pi/2) = 2$

$f(\pi) = 1$

$f'' = (\sin x + 1)(2\sin x) + (-2\cos x)(\cos x) = 0/DNE$

$2\sin^2 x + 2\sin x - 2\cos^2 x = 0$

$2(\sin^2 x + \sin x - (1 - \sin^2 x)) = 0$

$2(\sin^2 x + \sin x - 1 + \sin^2 x) = 0$

$2(2\sin^2 x + \sin x - 1) = 0$

$2\,\underbrace{(\sin x + 1)}_{=0}\,\underbrace{(2\sin x - 1)}_{=0} = 0$

$\sin x + 1 = 0$

$\sin x = -1$

$x = 3\pi/2 = -\pi/2$

$2\sin x - 1 = 0$

$2\sin x = 1$

$\sin x = 1/2$

$x = \pi/6,\ 5\pi/6$

f'': $-\pi$ − $-\frac{\pi}{2}$ − $\frac{\pi}{6}$ + $\frac{5\pi}{6}$ − π

critical pt(s):	$x = \pm \frac{\pi}{2}$
int. of inc:	$(-\pi, -\pi/2) \cup (\pi/2, \pi)$
int. of dec:	$(-\pi/2, \pi/2)$
extrema:	$x = -\pi$: rel min $x = \pi/2$: abs min $x = -\pi/2$: abs max $x = \pi$: rel max
infl. pt(s):	$x = \pi/6$
int. con. up:	$(\pi/6, 5\pi/6)$
int. con. down:	$(-\pi, \pi/6) \cup (5\pi/6, \pi)$

5. $f(x) = x^2 - x - \ln(x)$

$f'(x) = 2x - 1 - \frac{1}{x} = 0/DNE$

$2x \cdot \frac{x}{x} - 1 \cdot \frac{x}{x} - \frac{1}{x} = 0$

$\frac{2x^2 - x - 1}{x} = 0$

$\frac{(2x+1)(x-1)}{x} = 0$

$\boxed{x = \cancel{0}, \cancel{1}, -1/2}$ CPs

not in domain

f': DNE $-\frac{1}{2}$ 0 − 1 +

$f''(x) = 2 + \frac{1}{x^2} = 0/DNE$

$2 = -\frac{1}{x^2}$

$2x^2 = -1$

$x^2 = -1/2$

$x = \sqrt{-1/2}$ ✗

$\boxed{x = 0}$ POSSIBLE IP

f'': + 0 +

critical pt(s):	$x = 1$
int. of inc:	$(1, \infty)$
int. of dec:	$(0, 1)$
extrema:	$x = 1$: abs. minimum
infl. pt(s):	none
int. con. up:	$(-\infty, \infty)$
int. con. down:	none

6. $f(x) = x^2 \ln(x)$

$f'(x) = (\ln x)(2x) + (x^2)(\frac{1}{x}) = 0/DNE$

$2x\ln x + x = 0$

$\underbrace{x}\,(\underbrace{2\ln x + 1}) = 0$

$x = 0$

$2\ln x + 1 = 0$

$2\ln x = -1$

$\ln x = -1/2$

$\boxed{x = e^{-1/2} = \frac{1}{\sqrt{e}}}$ CPs

f': DNE 0 − $\frac{1}{\sqrt{e}}$ +

$f''(x) = (\ln x)(2) + (2x)(\frac{1}{x}) + 1 = 0/DNE$

$2\ln x + 2 + 1 = 0$

$2\ln x + 3 = 0$

$\ln x = -3/2$

$\boxed{x = e^{-3/2} = \frac{1}{e^{3/2}}}$ POSSIBLE IPs

f'': DNE 0 − $\frac{1}{e^{3/2}}$ +

critical pt(s):	$x = 0,\ e^{-1/2}$
int. of inc:	$(e^{-1/2}, \infty)$
int. of dec:	$(0, e^{-1/2})$
extrema:	$x = e^{-1/2}$: absolute max
infl. pt(s):	$x = e^{-3/2}$
int. con. up:	$(e^{-3/2}, \infty)$
int. con. down:	$(0, e^{-3/2})$