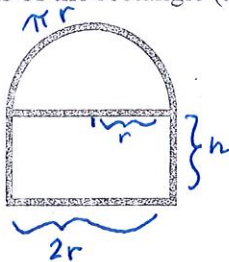


- 1) A Norman window has the outline of a semicircle on top of a rectangle as shown in the figure. Suppose there is  $8 + \pi$  feet of wood trim available for all 4 sides of the rectangle and the semicircle. Find the dimensions of the rectangle (and hence the semicircle) that will maximize the area of the window.



$$P = \overbrace{2h + 4r}^{\text{rectangle}} + \overbrace{\pi r}^{\text{semi-circle}} = \pi + 8$$

$$h = \frac{\pi + 8 - \pi r - 4r}{2}$$

$$A = \overbrace{2r(h)}^{\text{rectangle}} + \overbrace{\frac{1}{2}\pi r^2}^{\text{semi-circle}}$$

$$= 2r \left( \frac{\pi + 8 - \pi r - 4r}{2} \right) + \frac{1}{2}\pi r^2$$

$$= \pi r + 8r - \pi r^2 - 4r^2 + \frac{1}{2}\pi r^2$$

$$A' = \pi + 8 - 2\pi r - 8r + \pi r = 0$$

$$\pi + 8 - \pi r - 8r = 0$$

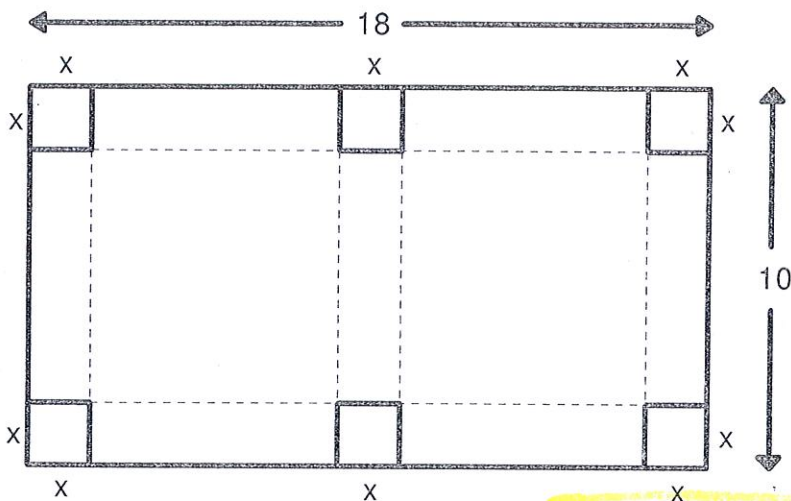
$$\pi + 8 = \pi r + 8r$$

$$\frac{\pi + 8}{\pi + 8} = \frac{r(\pi + 8)}{\pi + 8}$$

$$r = 1$$

$$h = 2$$

1) Pizza Hut has just introduced a larger personal size pizza. The Pizza Hut Company needs a design for a box to package this new pizza. The box is to be made from a piece of cardboard of size 10 inches by 18 inches. The company will generously award a design team that designs a box with the largest volume. The design must be simple, as shown below. Six squares of width  $x$  are to be cut from the cardboard, which will then be folded into a box. Your team is challenged to determine the dimensions of the box that will maximize its volume.



$$V = lwh$$

$$= (10 - 2x)(9 - 1.5x)(x)$$

$$= (10 - 2x)(9x - 1.5x^2)$$

$$V' = (9x - 1.5x^2)(-2) + (10 - 2x)(9 - 3x) = 0$$

$$= -18x + 3x^2 + 90 - 30x - 18x + 6x^2 = 0$$

$$9x^2 - 66x + 90 = 0$$

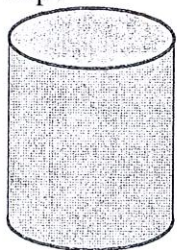
$$3(3x^2 - 22x + 30) = 0$$

$$x = \frac{22 \pm \sqrt{484 - 360}}{6} = \frac{22 \pm \sqrt{124}}{6} = \frac{x = 5.523}{+ 1.811}$$

$$V' \leftarrow \begin{matrix} + & - & + \\ & 1.811 & 5.523 \end{matrix}$$

$l = 6.38 \quad w = 6.285 \quad h = 1.81 \text{ in}$

2) Pizza Hut also wants to produce a right circular cylindrical cup (open top) that holds 265 cubic centimeters of liquid when filled to the top. The Pizza Hut Company wants your team to design a cup that can be constructed with the least amount of material, so they can cut back on production costs. What are the dimensions of the cup?



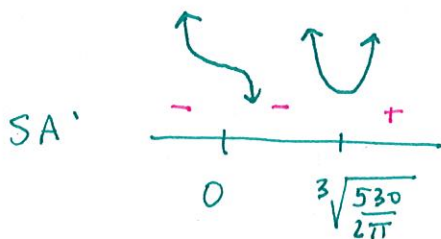
$$V = \pi r^2 h = 265$$

$$h = \frac{265}{\pi r^2}$$

$$SA = \pi r^2 + 2\pi r h$$

$$= \pi r^2 + 2\pi r \left( \frac{265}{\pi r^2} \right)$$

$$= \pi r^2 + \frac{530}{r}$$



$$SA' = 2\pi r - \frac{530}{r^2} = 0$$

$$\frac{530}{r^2} = 2\pi r$$

$$2\pi r^3 = 530$$

$$r^3 = \frac{530}{2\pi}$$

$h = \frac{265}{\pi \left( \frac{\sqrt[3]{530}}{\sqrt{2\pi}} \right)^2}$   
 $r = 7.4803$

$r = \sqrt[3]{\frac{530}{2\pi}}$   
 $r = 4.386$