# Related Rates Problem Set

1. If $z^2 = x^2 + y^2$, $dx/dt = 2$, and $dy/dt = 3$, find $dz/dt$ when $x = 5$ and $y = 12$. Assume that $z \geq 0$.

3.538

2. A particle moves along the curve $y = \sqrt{1 + x^3}$. As it reaches the point $(2, 3)$, the y-coordinate is increasing at a rate of 4 cm/sec. How fast is the x-coordinate of the point changing at this instant?

2

3. Suppose $PV = k$ where $k$ is a constant and $\frac{dP}{dt} = 2$ when $P = 4$ and $V = 5$. Find $\frac{dV}{dt}$.

$-\frac{5}{2}$

4. Two cars start moving from the same point. One travels south at 60 m/hr and the other travels west at 25 m/hr. At what rate is the distance between the cars increasing two hours later?

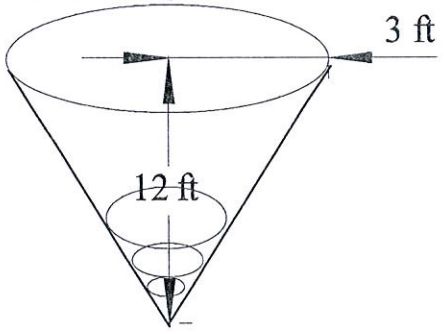
65

5. A plane flying horizontally at an altitude of 1 mile and a speed of 500 mph passes over a radar station. Find the rate at which the distance from the plane to the station is increasing when it is 2 miles away from the station.

433.013

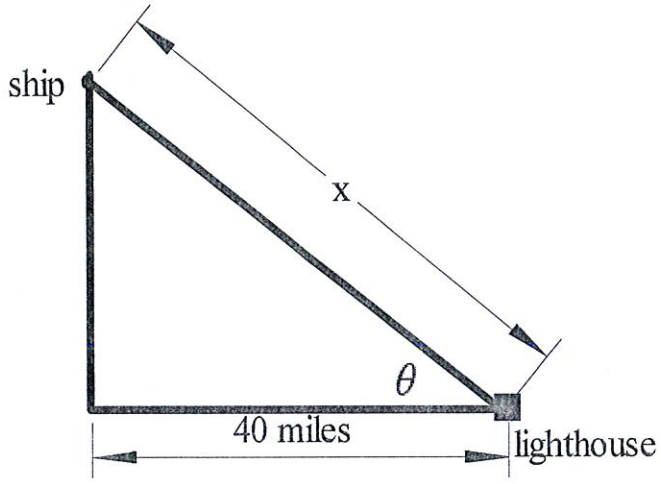
6. A conical water tower has a height of 12 ft and a radius of 3 ft. Water is pumped into the tank at a rate of 4 ft$^3$/min. How fast is the water level rising when the water level is 6 ft?
See the diagram.

0.566



7. A ship is 40 miles west of a lighthouse. The ship is heading north at a rate such that the angle $\theta$, shown in the diagram below, is changing at a constant rate of $0.7$ radians per hour. At what rate is the distance $x$ between the ship and the lighthouse changing when $\theta = 0.4$ radians?

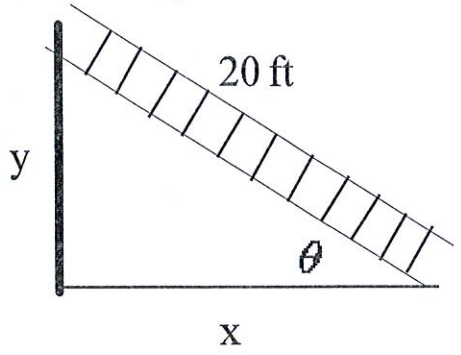
12.853




8. At noon, ship A is 100 km west of ship B. Ship A is sailing south at 35 km/hr and ship B is sailing north at 25 km/hr. How fast is the distance between the ships changing at 4:00 P.M.?

55.385

9. An 20 ft long ladder is leaning against a wall. The bottom of the ladder is sliding away from the wall at a rate of 2.5 ft/sec. See the diagram.




a) How fast is the top of the ladder sliding down the wall when $x = 12$ ft. Note that this rate is $dy/dt$ .

-1.875

b) How fast is the angle $\theta$ changing when $x = 12$ ft?

-0.156

c) How fast is the area of the triangle changing when $x = 12$ ft?

8.75

1. Eq: $z^2 = x^2 + y^2$

Der: $2z\frac{dz}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$

($z$: 13, find; $x$: 5, 2; $y$: 12, 3)

$x^2 + y^2 = z^2$

$5^2 + 12^2 = z^2$

$25 + 144 = z^2$

$z^2 = 169$

$z = 13$

Sub: $2(13)\frac{dz}{dt} = 2(5)(2) + 2(12)(3)$

$$\frac{dz}{dt} = \frac{20 + 72}{26} = \frac{92}{26} = 3.538$$

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2. Eq: $y = \sqrt{1 + x^3}$

Der: $\frac{dy}{dt} = \frac{1}{2}(1 + x^3)^{-1/2} \cdot 3x^2\frac{dx}{dt}$

($\frac{dy}{dt}$: 4; $x$: 2; $x$: 2; $\frac{dx}{dt}$: Find)

Sub: $4 = \frac{1}{2}(1 + 2^3)^{-1/2} \cdot 3(2)^2\frac{dx}{dt}$

$$\frac{dx}{dt} = \frac{4}{6(1+2^3)^{-1/2}} = \frac{4 \cdot 3}{6} = 2 \text{ cm/sec}$$

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3. Eq: $PV = K$

Der: $P\frac{dV}{dt} + V\frac{dP}{dt} = \frac{dK}{dt}$

($P$: 4; $\frac{dV}{dt}$: find; $V$: 5; $\frac{dP}{dt}$: 2; $\frac{dK}{dt}$: 0)

Sub: $4\frac{dv}{dt} + 5(2) = 0$

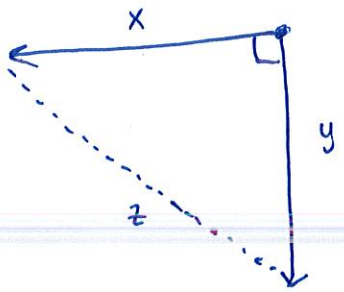
$$\frac{dv}{dt} = \frac{-10}{4} = -\frac{5}{2}$$

1.

Eq: $x^2 + y^2 = z^2$

Der: $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

$x$: 50, $\frac{dx}{dt}$: 25, $y$: 120, $\frac{dy}{dt}$: 60, $z$: 130, $\frac{dz}{dt}$: Find




Sub: $2(50)(25) + 2(120)(60) = 130\frac{dz}{dt}$

$2500 + 14400 = 2(130)\frac{dz}{dt}$

$$\frac{dz}{dt} = \frac{16900}{260} = 65 \text{ mph}$$

$x^2+y^2=z^2$

$50^2 + 120^2 = z^2$

$2500 + 14400 = z^2$

$z^2 = 16900$

$z = 130$

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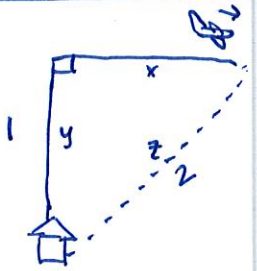
5. Eq: $x^2 + y^2 = z^2$

Der: $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

$x$: $\sqrt{3}$, $\frac{dx}{dt}$: 500, $y$: 1, $\frac{dy}{dt}$: 0, $z$: 2, $\frac{dz}{dt}$: Find




$x^2+y^2=z^2$

$x^2 + 1^2 = 2^2$

$x^2 = 4-1$

$x = \sqrt{3}$

Sub: $2(\sqrt{3})(500) + 2(1)(0) = 2(2)\frac{dz}{dt}$

$\sqrt{3}\cdot 500 = 2\,dz/dt$

$$dz/dt = \frac{\sqrt{3}\cdot 500}{2} = 433.013 \text{ mph}$$

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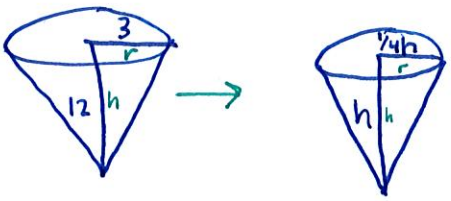
6. Eq: $V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi(\frac{1}{4}h)^2 h = \frac{\pi}{48}h^3$



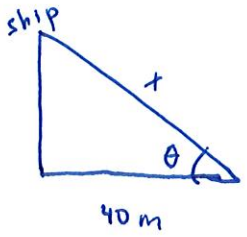

Der: $\frac{dV}{dt} = \frac{\pi}{16}h^2\frac{dh}{dt}$

$\frac{dV}{dt}$: 4, $h$: 6, $\frac{dh}{dt}$: Find

Sub: $4 = \frac{\pi}{16}(36)\frac{dh}{dt}$

$$\frac{dh}{dt} = \frac{4}{36\pi/16} = 0.566 \text{ ft/min}$$

7. Eq: $\cos\theta = \frac{40}{x}$




Der: $-\sin\theta \frac{d\theta}{dt} = -\frac{40}{x^2}\frac{dx}{dt}$

$\theta$: 0.4, $\frac{d\theta}{dt}$: 0.7, $x$: 43.428, $\frac{dx}{dt}$: Find

$\cos 0.4 = \frac{40}{x}$

$x = \frac{40}{\cos 0.4} = 43.428$

Sub: $-\sin(0.4)\cdot 0.7 = \frac{-40}{(43.428)^2}\cdot\frac{dx}{dt}$

$$\frac{dx}{dt} = \frac{-\sin 0.4 \cdot 0.7}{\frac{-40}{(43.428)^2}} = 12.853 \text{ rad/hr}$$

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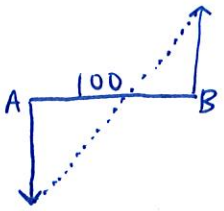
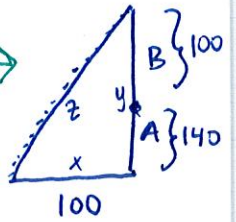
8. Eq: $x^2 + y^2 = z^2$




Redraw:




Der: $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

$x$: 100, $\frac{dx}{dt}$: 0, $y$: 240, $\frac{dy}{dt}$: 60, $z$: 260, $\frac{dz}{dt}$: Find

$x^2+y^2=z^2$

$100^2 + 240^2 = z^2$

$z^2 = \sqrt{67600}$

$z = 260$

Sub: $100(0) + 240(60) = 260\frac{dz}{dt}$

$$\frac{dz}{dt} = \frac{240(60)}{260} = 55.384 \text{ km/hr}$$

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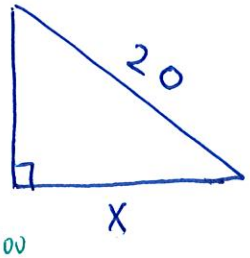
9. Eq: $x^2 + y^2 = z^2$

(a)

Der: $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

$x$: 12, $\frac{dx}{dt}$: 2.5, $y$: 16, $\frac{dy}{dt}$: Find, $z$: 20, $\frac{dz}{dt}$: 0




$x^2+y^2=z^2$

$12^2 + y^2 = 20^2$

$144 + y^2 = 400$

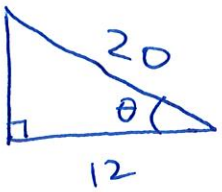
$y^2 = 256$

$y = 16$

Sub: $12(2.5) + 16\frac{dy}{dt} = 20(0)$

$$\frac{dy}{dt} = \frac{-12(2.5)}{16} = -1.875 \text{ ft/sec}$$

9. Eq: $\cos\theta = \frac{x}{20}$




$\cos\theta = \frac{12}{20}$

$\theta = \arccos\left(\frac{12}{20}\right)$

$\theta = 0.927$

(b)

Der: $-\sin\theta \frac{d\theta}{dt} = \frac{1}{20}\frac{dx}{dt}$

0.927 ↑ $\frac{d\theta}{dt}$ Find ↑ $\frac{dx}{dt}$ 2.5

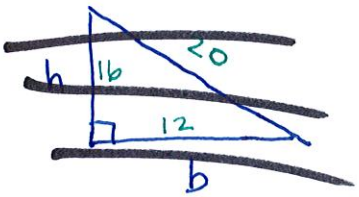
Sub: $-\sin(0.927)\cdot\frac{d\theta}{dt} = \frac{1}{20}(2.5)$

$$\frac{d\theta}{dt} = \frac{(\frac{1}{20})(2.5)}{-\sin(0.927)} = -0.156 \text{ rad/sec}$$

(c) ~~Eq: $A = \frac{1}{2}bh$~~

~~$A = \frac{1}{2}b\cdot\frac{4}{3}b$~~

~~$A = \frac{2}{3}b^2$~~




$h^2 + b^2 = 20^2$

$h^2 + 12^2 = 400$

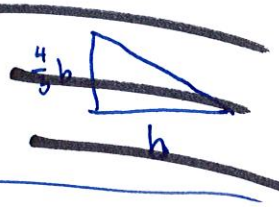
$h^2 = 256$

$h = 16$

~~Der: $\frac{dA}{dt} = \frac{2}{3}\cdot 2b\frac{db}{dt}$~~

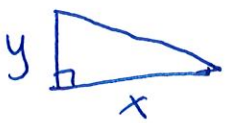
~~↑ Find ↑ 12 ↑ 2.5~~

~~To avoid product rule, get h in terms of b~~




~~Sub: $\frac{dA}{dt} = \frac{2}{3}\cdot 2\cdot 12\cdot 2.5 = 40$~~

(c) Eq: $A = \frac{1}{2}xy$




Der: $\frac{dA}{dt} = \frac{1}{2}x\frac{dy}{dt} + \frac{1}{2}y\frac{dx}{dt}$

12 ↑ $\frac{dy}{dt}$ -1.875 ↑ 16 ↑ 2.5

Sub: $\frac{dA}{dt} = \frac{1}{2}(12)(-1.875) + \frac{1}{2}(16)(2.5) = 8.75 \text{ ft}^2/\text{sec}$