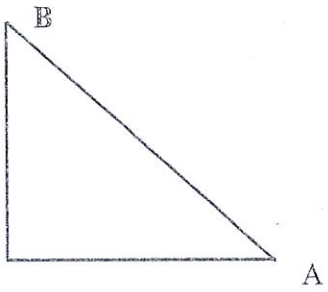
Practice Problems for Related Rates - AP Calculus BC

π

1. A circular plate of metal is heated in an oven, its radius increases at a rate of 0.01 cm/min. At what rate is the area of the plate increasing when the radius is 50 cm?

680

2. Two commercial jets at 40,000 ft are flying at 520 mi/hr along straight line courses that cross at right angles. How fast is the distance between them closing when the airplane A is 5 mi from the intersection point and airplane B is 12 mi from the intersection point?



-0.00006

3. A spherical tootsie roll pop that you are enjoying is giving up volume at a steady rate of 0.08 ml/min. How fast will the radius of the pop be decreasing when the tootsie pop is 20mm across?

0.0398

4. Assume that sand allowed to pour onto a level surface will form a pile in the shape of a cone, with its height equal to the diameter of the base. If sand is poured at 2 cubic meters per second, how fast is the height of the pile increasing when the base is 8 meters in diameter?

5. Water is flowing out at the rate of 50 cubic meters/min from a shallow conical reservoir (vertex downward) of base radius 45 m and height 6m.

-0.011
-0.0849

   a) How fast is the water level falling when the water is 5m deep?
   b) How fast is the radius of the water's surface changing then?

-0.0859

6. A funnel in the shape of an inverted cone is 30 cm deep and has a diameter across the top of 20 cm. Liquid is flowing out of the funnel at the rate of 12 cm^3/sec. At what rate is the height of the liquid decreasing at the instant when the liquid in the funnel is 20 cm deep?

7. [1982 AB 4] A ladder 15 feet long is leaning against a building so that end X is on level ground and end Y is on the wall. X is moved away from the building at the constant rate of 0.5 foot per second.

-0.375
1.3125

a) Find the rate in feet per second at which the height of the ladder above the ground is changing when X is 9 feet from the building.

b) find the rate of change in square feet per second of the area of the triangle formed by the building, the ground, and the ladder when X is 9 feet from the building.

8. [1984 AB5, BC2] A balloon is in the shape of a cylinder and has hemispherical ends of the same radius as that of the cylinder. (i.e., it looks like a medicine capsule). The balloon is being inflated at the rate of 261π cubic centimeters per minute. At the instant that the radius of the cylinder is 3 cm, the volume of the balloon is 144π cubic centimeters and the radius of the cylinder is increasing at the rate of 2 centimeters per minute.

2
5

a) At this instant, what is the height of the cylinder?

b) At this instant, how fast is the height of the cylinder changing?

9.

A boat is pulled into a dock by a rope attached to it and passing through a pulley on the dock positioned 5 meters higher than the boat. If the rope is being pulled in at a rate of 2 m.sec, how fast is the boat approaching the dock when it is 12 meters away from the dock?

-2.167

10. [1990 AB 4] The radius r of a sphere is increasing at the constant rate of 0.04 centimeters per second.

50.265

a) At the time when the radius of the sphere is 10 cm., what is the rate of increase of its volume?

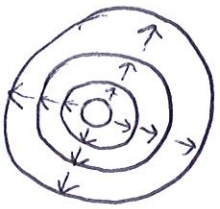
0.753

b) At the time when the volume of the sphere is 36π cubic centimeters, what is the rate of increase of the area of a cross section through the center of the sphere?

0.282

c) At the time when the volume and the radius of the sphere are increasing at the same numerical rate, what is the radius?

1. $[A = \pi r^2]$



$$\frac{dA}{dt} = \pi 2r \frac{dr}{dt}$$

Find ↑ $\frac{dA}{dt}$; 50 ↑ r; 0.01 ↑ $\frac{dr}{dt}$

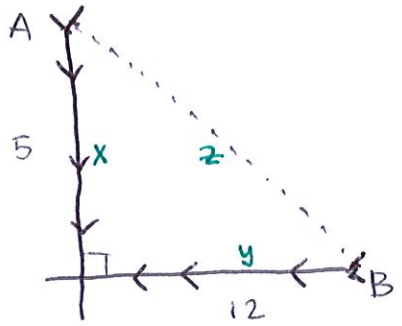
$$\frac{dA}{dt} = \pi(2)(50)(0.01) = \pi \text{ cm}^2/\text{min}$$

2. $[x^2 + y^2 = z^2]$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$$

$x = 5$, $\frac{dx}{dt} = 520$, $y = 12$, $\frac{dy}{dt} = 520$, $z = 13$, Find $\frac{dz}{dt}$

$$\frac{dz}{dt} = \frac{5(520) + 12(520)}{13} = 680 \text{ mph}$$

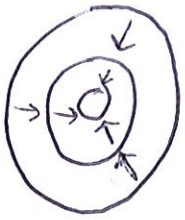


$5^2 + 12^2 = z^2$

$z^2 = 169$

$z = 13$

3. $[V = \frac{4}{3}\pi r^3]$



$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

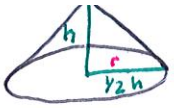
$\frac{dV}{dt} = -0.08$, $r = 10$, Find $\frac{dr}{dt}$

$$\frac{dr}{dt} = \frac{-0.08}{4\pi(10)^2} = -0.00006 \text{ ml/min}$$

4. $\left[V = \frac{1}{3}\pi r^2 h \right]$

$V = \frac{1}{3}\pi \left(\frac{1}{2}h\right)^2 h$

$V = \frac{\pi}{12} h^3$



height = diameter

height = 2 radius

radius = $\frac{1}{2}$ height

$\frac{dV}{dt} = \frac{\pi}{12} \cdot 3h^2 \frac{dh}{dt}$

$\frac{dV}{dt}$: 2 ; h: 8 ; $\frac{dh}{dt}$: Find

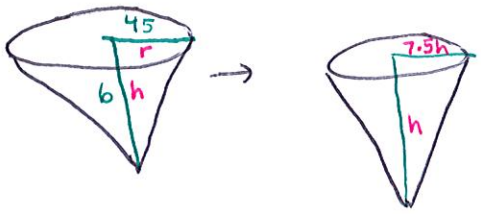
$2 = \frac{\pi}{12} \cdot 3(8)^2 \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{2}{\frac{\pi}{12} \cdot 3 \cdot 64} = 0.0398$ m/sec

5. A. $\left[V = \frac{1}{3}\pi r^2 h \right]$

$V = \frac{1}{3}\pi (7.5h)^2 h$

$V = 18.75\pi h^3$



$\frac{45}{6} = \frac{r}{h}$

$45h = 6r$

$r = \frac{45h}{6} = 7.5h$

$\frac{dV}{dt} = 56.25\pi h^2 \frac{dh}{dt}$

$\frac{dV}{dt}$: -50 ; h: 5 ; $\frac{dh}{dt}$: Find

$-50 = 56.25\pi (5)^2 \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{-50}{56.25\pi \cdot 25} = -0.011$ m/min

B. $\left[V = \frac{1}{3}\pi r^2 h \right]$

$\frac{dV}{dt} = \frac{1}{3}\pi \left(r^2 \frac{dh}{dt} + h \cdot 2r \frac{dr}{dt} \right)$

$\frac{dV}{dt}$: -50 ; r: 37.5 ; $\frac{dh}{dt}$: -0.011 ; h: 5 ; r: 37.5 ; $\frac{dr}{dt}$: Find

$-50 = \frac{1}{3}\pi \left[(37.5)^2 (-0.011) + (5)(2)(37.5) \frac{dr}{dt} \right]$

$\frac{dr}{dt} = -0.0849$ m/min

~~$r = \frac{45h}{6} = 7.5h$~~

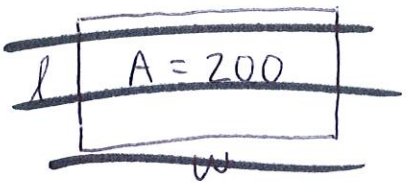
$r = 7.5h$

$r = 7.5(5) = 37.5$

6. $[A = \ell w]$

$\frac{dA}{dt} = \ell \frac{dw}{dt} + w \frac{d\ell}{dt}$

$\frac{dA}{dt}$: Find; ℓ: 200; $\frac{dw}{dt}$: -0.5; w: 4



~~$A = \ell w$~~

~~200~~

6. $V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi (\frac{1}{3}h)^2 h$

$[V = \frac{1}{27}\pi h^3]$

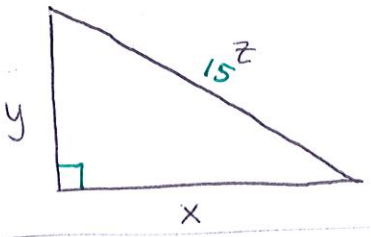
$\frac{dV}{dt} = \frac{1}{9}\pi h^2 \frac{dh}{dt}$

$\frac{dV}{dt}$: -12; h: 20; $\frac{dh}{dt}$: Find

$-12 = \frac{1}{9}\pi (20)^2 \frac{dh}{dt}$

$\frac{dh}{dt} = -0.0859$ cm/sec

7. $[x^2 + y^2 = z^2]$



$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

x: 9; $\frac{dx}{dt}$: 0.5; y: 12; $\frac{dy}{dt}$: Find; z: 15; $\frac{dz}{dt}$: 0

$(9)(0.5) + (12)\frac{dy}{dt} = (15)(0)$

$\frac{dy}{dt} = \frac{-(9)(0.5)}{12} = -0.375$ ft/sec

$x^2 + y^2 = z^2$

$9^2 + y^2 = 15^2$

$y^2 = 144$

$y = 12$

B. $[A = \frac{1}{2} x y]$

$\frac{dA}{dt} = \frac{1}{2}x\frac{dy}{dt} + \frac{1}{2}y\frac{dx}{dt}$

$\frac{dA}{dt}$: Find; x: 9; $\frac{dy}{dt}$: -0.375; y: 12; $\frac{dx}{dt}$: 0.5

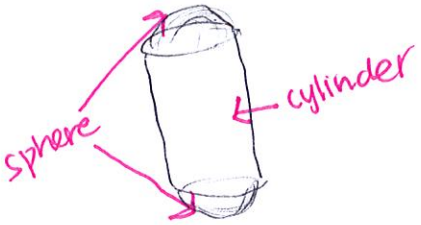
$\frac{dA}{dt} = \frac{1}{2}(9)(-0.375) + \frac{1}{2}(12)(0.5) = 1.3125$ ft^2/sec

8. $\left[V = \pi r^2 h + \frac{4}{3}\pi r^3 \right]$

(V: 144π; r: 3; r^3: 3)

A. $144\pi = \pi (3)^2 h + \frac{4}{3}\pi (3)^3$

$h = 12$ cm



B. $\left[V = \pi r^2 h + \frac{4}{3}\pi r^3 \right]$

$$\frac{dV}{dt} = \pi \left[r^2 \frac{dh}{dt} + h \cdot 2r \frac{dr}{dt} \right] + 4\pi r^2 \frac{dr}{dt}$$

($\frac{dV}{dt}$: 261π; r: 3; $\frac{dh}{dt}$: Find; h: 12; r: 3; $\frac{dr}{dt}$: 2; r: 3; $\frac{dr}{dt}$: 2)

$$261\pi = \pi \left[(3)^2 \frac{dh}{dt} + (12)(2)(3)(2) \right] + 4\pi (3)^2 (2)$$

$\frac{dh}{dt} = 5$ cm/min

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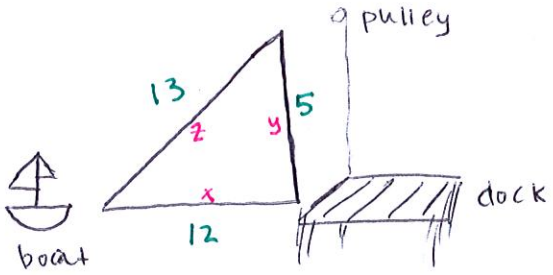
a. $\left[x^2 + y^2 = z^2 \right]$

$$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 2z \frac{dz}{dt}$$

(x: 12; $\frac{dx}{dt}$: Find; y: 5; $\frac{dy}{dt}$: 0; z: 13; $\frac{dz}{dt}$: −2)



$$12 \frac{dx}{dt} + 5(0) = 13(-2)$$

$$\frac{dx}{dt} = \frac{13(-2)}{12} = -2.167 \text{ m/sec}$$

$x^2 + y^2 = z^2$

$12^2 + 5^2 = z^2$

$144 + 25 = z^2$

$z = 13$

10. $\left[V = \frac{4}{3}\pi r^3\right]$

A. $\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

↑ Find ↑ 10 ↑ 0.04

$\frac{dV}{dt} = 4\pi(10)^2 \cdot 0.04 =$ **50.265 cm/sec**

B. $V = \frac{4}{3}\pi r^3$

$36\pi = \frac{4}{3}\pi r^3$

$r = \sqrt[3]{27} = 3$

$\left[A = \pi r^2\right]$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

↑ Find ↑ 3 ↑ 0.04

$\frac{dA}{dt} = 2\pi(3)(0.04) = 0.753$ cm²/sec

C. $V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$1 = 4\pi r^2$

$r^2 = \frac{1}{4\pi}$

$r = \sqrt{1/4\pi} =$ **0.282 cm/sec**