Calculus Name________________________________ ID: 1

Curve Sketching

Date________________ Period____

For each problem, find the: x and y intercepts, x-coordinates of the critical points, open intervals where the function is increasing and decreasing, x-coordinates of the inflection points, open intervals where the function is concave up and concave down, and relative minima and maxima. Using this information, sketch the graph of the function.

✓ 1) $y = 2x^3 + 4x^2 + 2x$

x-int:

$0 = 2x^3 + 4x^2 + 2x$

$0 = 2x(x^2 + 2x + 1)$

$0 = 2x(x+1)^2$

$x = 0, -1$

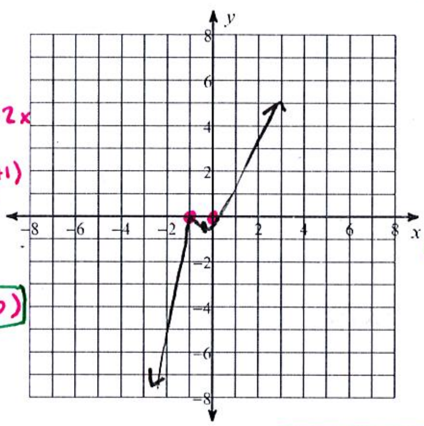
$\boxed{(0,0) + (-1,0)}$

y-int:

$y = 0 + 0 + 0$

$= 0$

$\boxed{(0,0)}$



$y' = 6x^2 + 8x + 2 = 0/DNE$

$2(6x^2 + 4x + 1) = 0$

$2(3x+1)(x+1) = 0$

$x = -1/3, -1$

y': $+$ $-$ $+$; -1, $-\frac{1}{3}$

$y'' = 12x + 8 = 0/DNE$

$4(3x+2) = 0$

$x = -2/3$

y'': $-$ $+$; $-\frac{2}{3}$

critical pt(s): $x = -1/3 + -1$
int. of inc: $(-\infty, -\frac{1}{3}) \cup (-1, \infty)$
int. of dec: $(-\frac{1}{3}, 1)$
extrema: $x = -1/3$ rel min; $x = -1$ rel max
infl. pt(s): $x = -2/3$
int. con. up: $(-2/3, \infty)$
int. con. down: $(-\infty, -2/3)$

× 2) $\cancel{y = -\frac{x^3}{x^2 - 1}}$

x-int:

$0 = \frac{-x^3}{x^2-1}$

$0 = -x^3$

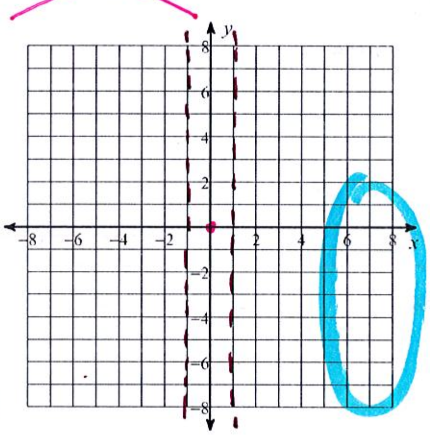
$x = 0$

$\boxed{(0,0)}$

y-int:

$y = \frac{-0^3}{0^2-1} = 0$

$\boxed{(0,0)}$



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$y' = \frac{(x^2-1)(-3x^2) - (-x^3)(2x)}{(x^2-1)^2} = 0/DNE$

$\frac{-3x^4 + 3x^2 + 2x^4}{(x^2-1)^2} = 0/DNE$

$\frac{-x^4 + 3x^2}{(x^2-1)^2} = 0/DNE$

$\frac{-x^2(x^2-3)}{(x^2-1)^2} = 0$

$x = 0, \pm\sqrt{3}, \cancel{\pm 1}$ not in domain

y': $-\sqrt{3}$, 0, $\sqrt{3}$

$y'' = \frac{(x^2-1)^2(-4x^3+6x) - (-x^4+3x^2)(2(x^2-1)\cdot 2x)}{(x^2-1)^4}$

$\frac{(x^2-1)\left[(x^2-1)(-4x^3+6x) - (-x^4+3x^3)(4x)\right]}{(x^2-1)^4}$

x 3) ~~$f(x)=\frac{x^3}{12}-\frac{x^2}{6}-\frac{x}{3}$~~

OMIT

x-int:

$0=\frac{x^3}{12}-\frac{x^2}{6}-\frac{x}{3}$

$0=x^3-2x^2-4x$

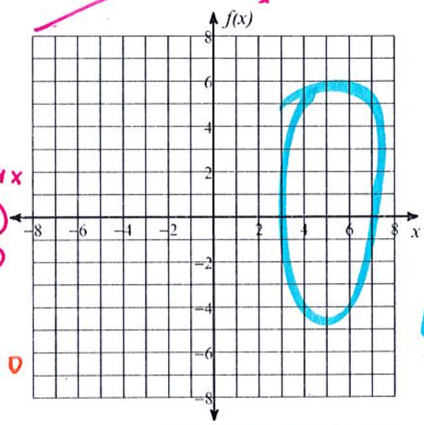
$0=x(x^2-2x-4)$

$0=x(\cancel{x-2}\quad)$

y-int:

$y=0^3-x^2-0=0$

$(0,0)$



$f'=\frac{1}{4}x^2-\frac{1}{3}x-\frac{1}{3}=0/DNE$

$3x^2-4x-4=0$

$(3x+2)(x-2)=0$

$x=-2/3,\ 2$

f' $-\frac{2}{3}$ 2

$f''=6x-4=0/DNE$

$2(3x-2)=0$

$x=2/3$

f'' $-$ $\frac{2}{3}$ $+$

✓4) $f(x)=-(5x+30)^{\frac{1}{2}}$

x-int:

$0=-(5x+30)^{1/2}$

$0=(5x+30)^{1/2}$

$0=5x+30$

$-30=5x$

$x=-6$

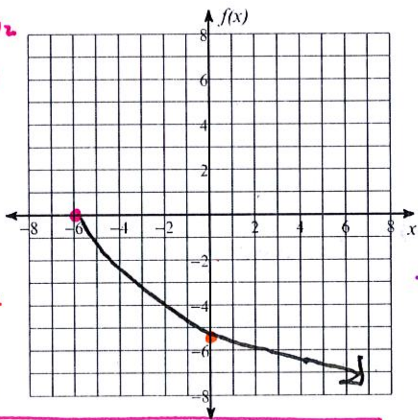
$\boxed{(-6,0)}$

y-int:

$y=-(0+30)^{1/2}$

$=-\sqrt{30}$

$\boxed{(0,-\sqrt{30})}$



$f'=-\frac{1}{2}(5x+30)^{-1/2}\cdot 5=0/DNE$

$\frac{-5}{2\sqrt{5x+30}}=0/DNE$

$x=6$

f' DNE -6 $-$

$f''=\frac{5}{2}(5x+30)^{-3/2}\cdot 5=0/DNE$

$\frac{25}{2(5x+30)^{-3/2}}=0/DNE$

$x=-6$

f'' DNE -6 $+$

critical pt(s):	$x=-6$
int. of inc:	none
int. of dec:	$(-6,\infty)$
extrema:	none
infl. pt(s):	none
int. con. up:	$(-6,\infty)$
int. con. down:	none

✓5) $y=-x^3-3x^2$

x-int:

$0=-x^3-3x^2$

$0=\cancel{x}-x^2(x+3)$

$x=0,-3$

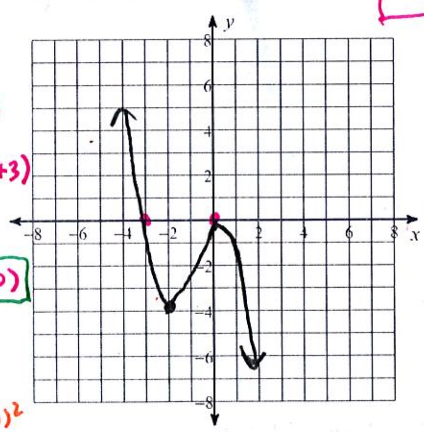
$\boxed{(0,0)+(-3,0)}$

y-int:

$y=-0^3-3(0)^2$

$y=0$

$\boxed{(0,0)}$



$f'=-3x^2-6x=0/DNE$

$-3x(x+2)=0$

$x=0,-2$

f' $-$ ∪ $+$ ∩ $-$; -2 0

$f''=-6x-6=0/DNE$

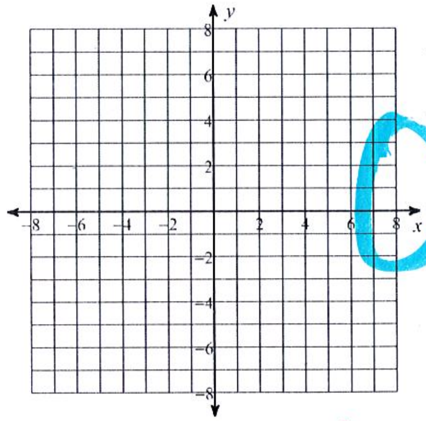
$-6(x+1)=0$

$x=-1$

f'' $+$ -1 $-$

critical pt(s):	$x=-2,0$
int. of inc:	$(-2,0)$
int. of dec:	$(-\infty,-2)\cup(0,\infty)$
extrema:	$x=-2$ rel min $x=0$ rel max
infl. pt(s):	$x=-1$
int. con. up:	$(-\infty,-1)$
int. con. down:	$(-1,\infty)$

× 6) ~~$y = -\frac{3}{16}(x+2)^{\frac{4}{3}} + \frac{3}{2}(x+2)^{\frac{1}{3}}$~~



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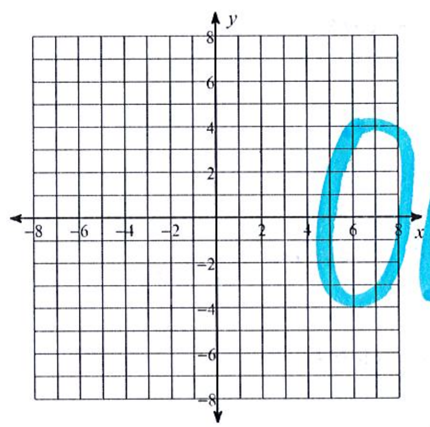
× 7) ~~$y = \frac{3}{16}(x-2)^{\frac{4}{3}} + \frac{3}{2}(x-2)^{\frac{1}{3}}$~~



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✓ 8) $f(x) = (-x+5)^{\frac{1}{2}}$

x-int:

$0 = (-x+5)^{1/2}$

$0 = -x+5$

$x = 5$

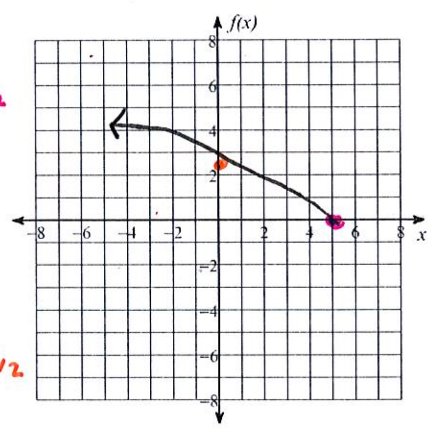
$\boxed{(5,0)}$

y-int:

$y = (-0+5)^{1/2}$

$y = \sqrt{5}$

$\boxed{(0, \sqrt{5})}$



$f' = \frac{1}{2}(-x+5)^{-1/2} \cdot -1 = 0/DNE$

$-\frac{1}{2} \cdot \frac{1}{\sqrt{-x+5}} = 0/DNE$

$x = 5$

f' — DNE, 5

$f'' = -\frac{1}{2} \cdot -\frac{1}{2}(-x+5)^{-3/2} \cdot -1 = 0/DNE$

$-\frac{1}{4}(-x+5)^{-3/2} = 0/DNE$

$-\frac{1}{4} \cdot \frac{1}{\sqrt{(-x+5)^3}} = 0/DNE$

$x = 5$

f'' — DNE, 5

critical pt(s):	$x = 5$
int. of inc:	none
int. of dec:	$(-\infty, 5)$
extrema:	none
infl. pt(s):	none
int. con. up:	none
int. con. down:	$(-\infty, 5)$

9) $f(x) = -\frac{x^3}{12} - \frac{x^2}{4}$

x-int:

$0 = -\frac{x^3}{12} - \frac{x^2}{4}$

$0 = -x^3 - 3x^2$

$0 = -x^2(x+3)$

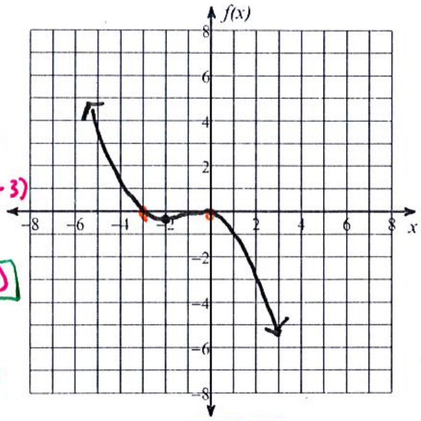
$x = 0, -3$

$\boxed{(0,0) + (-3,0)}$

y-int:

$y = 0 - 0 = 0$

$\boxed{(0,0)}$



$f' = -\frac{1}{4}x^2 - \frac{1}{2}x = 0/DNE$

$-\frac{1}{4}x(x+2) = 0$

$x = 0, -2$

f': $-$ | -2 | $+$ | 0 | $-$

$f'' = -\frac{1}{2}x - \frac{1}{2} = 0/DNE$

$-\frac{1}{2}(x+1) = 0$

$x = -1$

f'': $+$ | -1 | $-$

critical pt(s):	$x = -2, 0$
int. of inc:	$(-2, 0)$
int. of dec:	$(-\infty, -2) \cup (0, \infty)$
extrema:	$x = -2$ rel min $x = 0$ rel max
infl. pt(s):	$x = -1$
int. con. up:	$(-\infty, -1)$
int. con. down:	$(-1, \infty)$

10) $f(x) = -(5x+5)^{\frac{1}{3}}$

x-int:

$0 = -(5x+5)^{1/3}$

$0 = 5x + 5$

$-5 = 5x$

$x = -1$

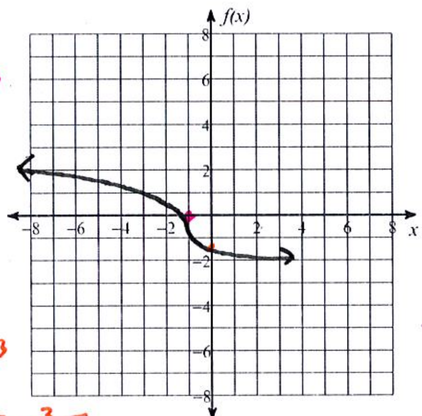
$\boxed{(-1, 0)}$

y-int:

$y = -(0+5)^{1/3}$

$y = -(5)^{1/3} = -\sqrt[3]{5}$

$\boxed{(0, -\sqrt[3]{5})}$



$f' = -\frac{1}{3}(5x+5)^{-2/3} \cdot 5 = 0/DNE$

$-\frac{5}{3} \cdot \frac{1}{(5x+5)^{2/3}} = 0/DNE$

$x = -1$

f': $-$ | -1 | $-$

$f'' = -\frac{5}{3} \cdot -\frac{2}{3}(5x+5)^{-5/3} = 0/DNE$

$\frac{10}{3} \cdot \frac{1}{(5x+5)^{5/3}} = 0/DNE$

$x = -1$

f'': $-$ | -1 | $+$

critical pt(s):	$x = -1$
int. of inc:	none
int. of dec:	$(-\infty, \infty)$
extrema:	none
infl. pt(s):	$x = -1$
int. con. up:	$(-1, \infty)$
int. con. down:	$(-\infty, -1)$