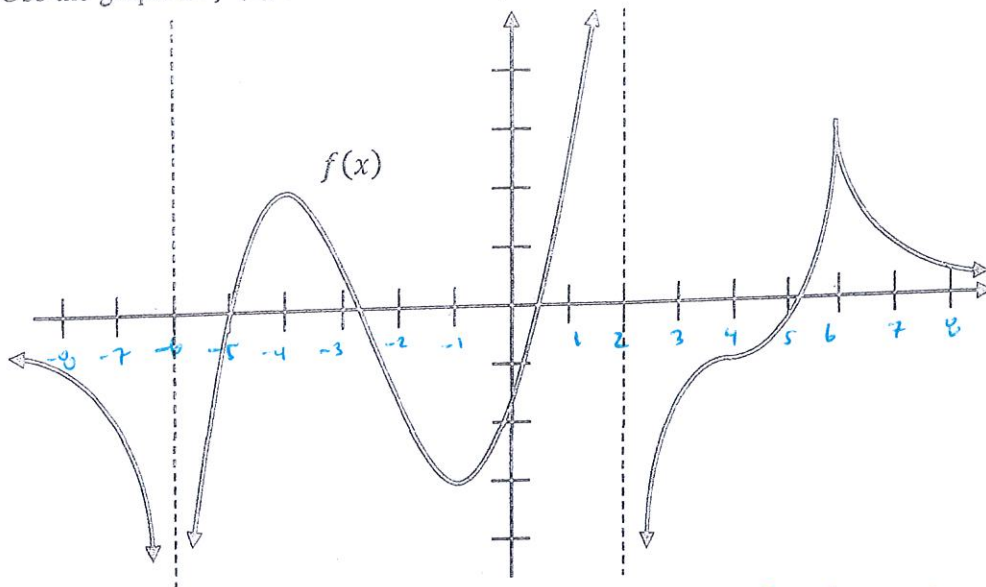


ANSWERS

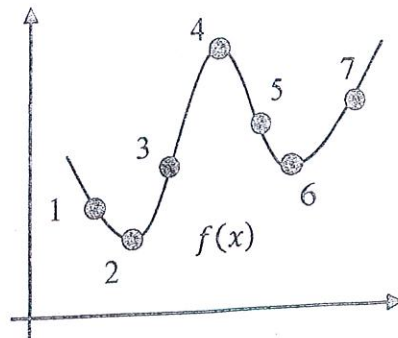
Worksheet 10 -- Increasing/Decreasing, Concavity, and Curve Sketching

1. Use the graph of $f(x)$ below to answer parts (a) through (h).



- (a) Find the intervals where $f(x)$ is increasing. $(-6, -4) \cup (-1, 2) \cup (2, 6)$
- (b) Find the intervals where $f(x)$ is decreasing. $(-\infty, -6) \cup (-4, -1) \cup (6, \infty)$
- (c) Find all the x -values where the slope of $f(x)$ is zero. $x = -4, -1, 4$
- (d) Find all the x -values where the derivative of $f(x)$ does not exist. $x = 6$ ($x = -6, 2$ ok)
- (e) Find all the critical points of $f(x)$. $x = -4, -1, 4, 6$ ($x = -6, 2$ ok)
- (f) Find the coordinates of all the relative maxima of $f(x)$. $(-4, 2) + (6, 3)$
- (g) Find the coordinates of all the relative minima of $f(x)$. $(-1, -3)$
- (h) Find all the x -values where $f(x)$ changes from increasing to decreasing or decreasing to increasing. $x = -4, -1, \pm 6$

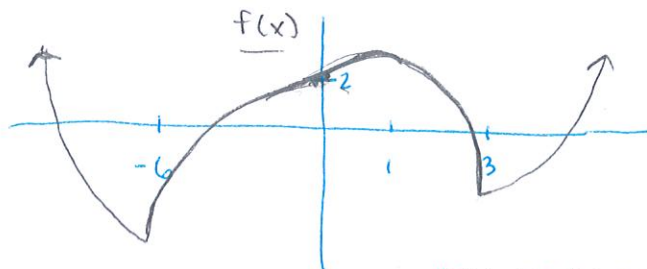
2. Use the numbered points on the graph of $f(x)$ at the right to answer parts (a) through (h).



- (a) At which points is the function increasing? 3, 7
- (b) At which points is there a relative maximum? 4
- (c) At which points is the slope negative? 1, 5
- (d) At which points is the slope zero? 2, 4, 6
- (e) At which points is there a relative minimum? 6
- (f) At which points is the function decreasing? 1, 5
- (g) At which points is the slope positive? 3, 7

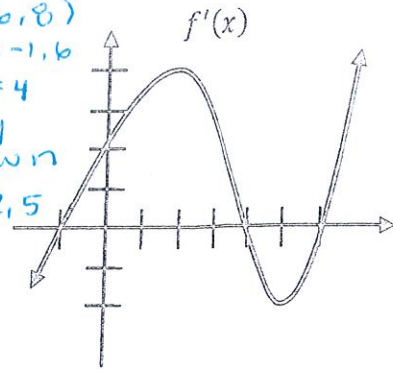
3. Sketch the graph of a single continuous function with a y -intercept at $(0, 2)$ that has the following properties.

- (1) $f'(x) < 0$ on $(-\infty, -6) \cup (1, 3)$
- (2) $f'(x) > 0$ on $(-6, 1) \cup (3, \infty)$
- (3) $f''(x) > 0$ on $(-\infty, -6) \cup (3, \infty)$
- (4) $f''(x) < 0$ on $(-6, 3)$



5. Use the graph of $f'(x)$ at the right to answer parts (a) through (i).

- (a) What is the slope of $f(x)$ at $x = 2$? **4**
- (b) For which x -values does $f(x)$ have a horizontal tangent line? **$x = -1, 4, 6$**
- (c) Find the intervals where $f(x)$ is increasing. **$(-1, 4) \cup (6, 8)$**
- (d) Find the x -values where $f(x)$ has a relative minimum. **$x = -1, 6$**
- (e) Find the x -values where $f(x)$ has a relative maximum. **$x = 4$**
- (f) Is $f(x)$ increasing or decreasing at $x = 5$? **decreasing**
- (g) Is $f(x)$ concave up or concave down at $x = 4$? **con. down**
- (h) Find the x -values where $f(x)$ has an inflection point. **$x = 2, 5$**
- (i) Find $\lim_{x \rightarrow -\infty} f(x)$. **∞**



6. Match each observation 1-5 to each conclusion A-E.

- (1) The point (3,4) is on the graph of $f'(x)$.
- (2) The point (3,4) is on the graph of $f(x)$.
- (3) The point (3,4) is on the graph of $f''(x)$.
- (4) The point (3,0) is on the graph of $f'(x)$ and (3,4) is on the graph of $f''(x)$.
- (5) The point (3,0) is on the graph of $f'(x)$ and (3, -4) is on the graph of $f''(x)$.

- (A) $f(x)$ has a relative minimum at $x = 3$. **4**
- (B) At $x = 3$, the graph of $f(x)$ is concave up. **3**
- (C) At $x = 3$, the tangent line to the graph of $f(x)$ has slope 4. **1**
- (D) At $x = 3$, the value of $f(x)$ is 4. **2**
- (E) $f(x)$ has a relative maximum at $x = 3$. **5**

7. In the graph of $f(x)$ at the right, assume that $(-1,1)$ is an inflection point. Use the graph to answer parts (a) through (k).

- (a) Find the intervals where $f(x)$ is decreasing. **$(-3, 1)$**
- (b) Find the intervals where $f'(x) > 0$. **$(-\infty, -3) \cup (1, \infty)$**
- (c) Find all the critical points. **$x = -3, 1$**
- (d) Find the intervals where $f(x)$ is concave up. **$(-1, 2)$**
- (e) Find the intervals where $f(x)$ is concave down. **$(-\infty, 1)$**
- (f) Is $f'(-4)$ positive, negative or zero? **positive**
- (g) Is $f''(-4)$ positive, negative, or zero? **negative**
- (h) Is $f'(0)$ positive, negative or zero? **negative**
- (i) Is $f''(0)$ positive, negative or zero? **positive**
- (j) Is $f'(1)$ positive, negative, or zero? **zero**
- (k) Is $f''(1)$ positive, negative, or zero? **positive**

