# Optimization AND RR combined

(1) A closed box with a square base is to have a volume of $10 \text{ m}^3$. The base costs $\$4/\text{m}^2$, the sides cost $\$2/\text{m}^2$ and the top $\$1/\text{m}^2$. What dimensions will give the minimum cost to build the box?

(2) A block of ice, in the shape of a right circular cylinder, is melting in such a way that both its height and its radius are decreasing at the rate of 1 cm/hr. How fast is the volume decreasing when the radius is 1 cm and the volume is $10\pi \text{ cm}^3$?

(3) . A 6-ft man walks away from a 15-ft lamppost at a speed of 3 ft/s. Find the rate at which his shadow is increasing in length.

4 A rectangular page is to contain 24 square inches of printable area. The margins at the top and bottom of the page are each 1 inch, one side margin is 1 inch, and the other side margin is 2 inches. What should the dimensions of the page be so that the least amount of paper is used?

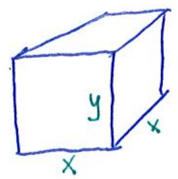
5 A plane flies directly over a man and is 6 kilometres above his head. The distance from the plane to the man is increasing at the rate of 400 kilometres per hour when the distance from the plane to the man is 10 kilometers. How fast is the plane moving?

6 A farmer wants to fence a rectangular plot of land with area 2400 $m^2$. She wants to keep her horses on one side of the field, so she plans to use additional fencing to build an internal divider, parallel to two sides of the fence. Wood for the outer walls costs \$3 per metre, and wood for the internal divider costs \$2 per metre. What is the minimum cost of the project?

# Optimization AND RR combined

Opt ①

A closed box with a square base is to have a volume of 10 m$^3$. The base costs \$4/m$^2$, the sides cost \$2/m$^2$ and the top \$1/m$^2$. What dimensions will give the minimum cost to build the box?




$V = x^2 y = 10$

so $y = \frac{10}{x^2}$

$SA = 4(x^2) + 1(x^2) + 2(4xy)$

$SA = 4x^2 + x^2 + 8x(y)$ → $\frac{10}{x^2}$

Eq: $SA = 5x^2 + \frac{80}{x}$

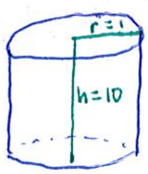
$SA' = 10x - \frac{80}{x^2} = 0$

critical points: $\boxed{x = \cancel{0},\ \sqrt[3]{8}} = 2$


The dimensions that minimize the cost are $2 \times 2 \times 5/2$ m

RR ②

A block of ice, in the shape of a right circular cylinder, is melting in such a way that both its height and its radius are decreasing at the rate of 1 cm/hr. How fast is the volume decreasing when the radius is 1 cm and the volume is $10\pi$ cm$^3$?




Eq: $V = \pi r^2 h$

$10\pi = \pi (1)^2 h$

$h = 10$

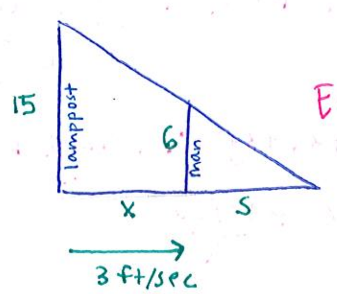
per: $\frac{dV}{dt} = \pi \left[ r^2 \frac{dh}{dt} + h \cdot 2r \frac{dr}{dt} \right]$

($\frac{dV}{dt}$: Find; $r$: 1; $\frac{dh}{dt}$: -1; $h$: 10; $r$: 1; $\frac{dr}{dt}$: -1)

$\frac{dV}{dt} = \pi (1^2 \cdot -1 + 10 \cdot 2 \cdot 1 \cdot -1) = -21\pi$ cm$^3$/hr

RR③

A 6-ft man walks away from a 15-ft lamppost at a speed of 3 ft/s. Find the rate at which his shadow is increasing in length.




Eq: $\frac{15}{x+s} = \frac{6}{s}$

$15s = 6x + 6s$

$9s = 6x$

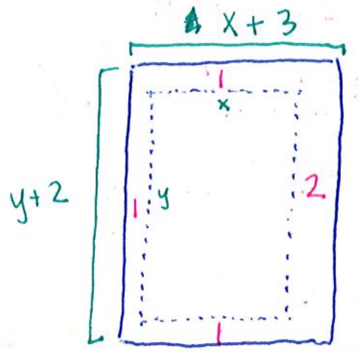
per: $9 \frac{ds}{dt} = 6 \frac{dx}{dt}$ → $9 \frac{ds}{dt} = 18$

($\frac{dx}{dt}$: 3)

$\boxed{\frac{ds}{dt} = 2 \text{ ft/sec}}$

Opt (4)

A rectangular page is to contain 24 square inches of printable area. The margins at the top and bottom of the page are each 1 inch, one side margin is 1 inch, and the other side margin is 2 inches. What should the dimensions of the page be so that the least amount of paper is used?




$xy = 24$

so $y = \frac{24}{x}$

$A = (x+3)(y+2)$   ($y = 24/x$)

$A = (x+3)\left(\frac{24}{x}+2\right)$

$A = 24 + 2x + \frac{72}{x} + 6$

Eq: $A = 30 + 2x + 72/x$

$A' = 2 - 72/x^2 = 0$

$x^2 = 36$

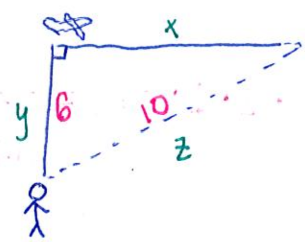
critical points $\boxed{x = \cancel{0}, \cancel{-6}, 6}$

+ − − + ; −6 0 6

The dim. w/ least paper is 9 x 6.

RR (5)

A plane flies directly over a man and is 6 kilometres above his head. The distance from the plane to the man is increasing at the rate of 400 kilometres per hour when the distance from the plane to the man is 10 kilometers. How fast is the plane moving?




Eq: $[x^2 + y^2 = z^2$

$x^2 + 6^2 = 10^2$

$x^2 = 64$

$x = 8$

Der: $[\cancel{2}x\frac{dx}{dt} + \cancel{2}y\frac{dy}{dt} = \cancel{2}z\frac{dz}{dt}$

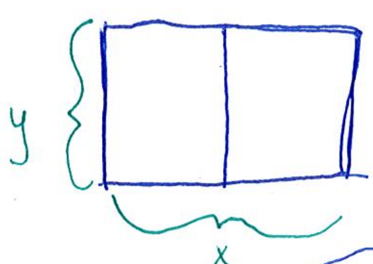
8, Find, 6, 0, 10, 400

$8\frac{dx}{dt} + 0 = 4000$

$\frac{dx}{dt} = \frac{4000}{8} = \boxed{500\ \frac{KM}{hr}}$

Opt (6)

A farmer wants to fence a rectangular plot of land with area 2400 $m^2$. She wants to keep her horses on one side of the field, so she plans to use additional fencing to build an internal divider, parallel to two sides of the fence. Wood for the outer walls costs \$3 per metre, and wood for the internal divider costs \$2 per metre. What is the minimum cost of the project?




$A = xy = 2400$

so $y = \frac{2400}{x}$

internal wall

$P = 3(x) + 3(x) + 3(y) + 3(y) + 2(y)$   ($y = 2400/x$)

$P = 3x + 3x + \frac{7200}{x} + \frac{7200}{x} + \frac{4800}{x}$

Eq: $P = 6x + \frac{19200}{x}$

$P' = 6 - \frac{19200}{x^2} = 0$

$x^2 = 3200$

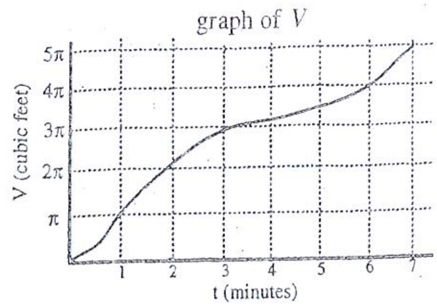
critical points $\boxed{x = 56.569, \cancel{0}}$

+ − + ; 0 56.569

The dimensions w/ min cost are 56.569 x $\frac{2400}{56.569}$ m.

The min cost is \$678.83.

6. Sand is being dumped on a pile in such a way that it always forms a cone whose base radius is always 3 times its height. The function $V$ whose graph is sketched in the figure gives the volume of the conical sand pile, $V(t)$, measured in cubic feet, after $t$ minutes. $\left(V(t) = \frac{1}{3}\pi r^2 h\right)$ At what approximate rate is the radius of the base changing after 6 minutes?




(A) 0.22 ft/min (B) 0.28 ft/min (C) 0.34 ft/min (D) 0.40 ft/min (E) 0.46 ft/min

Ans

3. Oil is leaking from a pipeline on the surface of a lake and forms an oil slick whose volume increases at a constant rate of 2000 cubic centimeters per minute. The oil slick takes the form of a right circular cylinder with both its radius and height changing with time. (Note: The volume $V$ of a right circular cylinder with radius $r$ and height $h$ is given by $V = \pi r^2 h$.)

(a) At the instant when the radius of the oil slick is 100 centimeters and the height is 0.5 centimeter, the radius is increasing at the rate of 2.5 centimeters per minute. At this instant, what is the rate of change of the height of the oil slick with respect to time, in centimeters per minute?

(b) A recovery device arrives on the scene and begins removing oil. The rate at which oil is removed is $R(t) = 400\sqrt{t}$ cubic centimeters per minute, where $t$ is the time in minutes since the device began working. Oil continues to leak at the rate of 2000 cubic centimeters per minute. Find the time $t$ when the oil slick reaches its maximum volume. Justify your answer.

10. When the area of an expanding square, in square units, is increasing three times as fast as its side is increasing, in linear units, the side is

(A) $\frac{2}{3}$ (B) $\frac{3}{2}$ (C) 3 (D) 2 (E) 1

Ans

11. What is the area of the largest rectangle that can be inscribed under the graph of $y = 2\cos x$ for $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$?

(A) 2.20 units$^2$

(B) 2.24 units$^2$

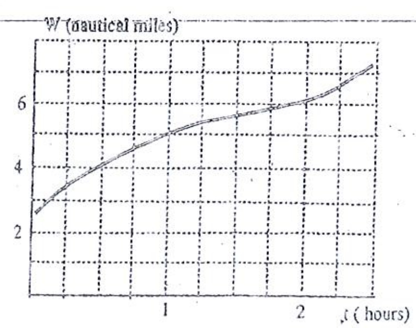
(C) 2.28 units$^2$

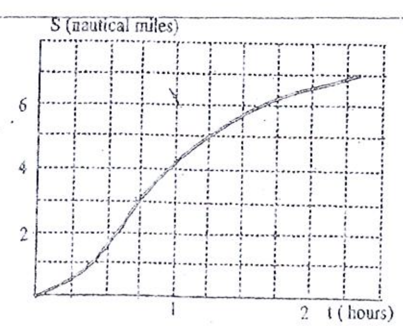
(D) 2.32 units$^2$

(E) 2.36 units$^2$

Ans

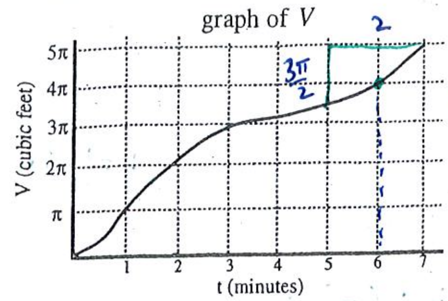
12. One ship traveling west is $W(t)$ nautical miles west of a lighthouse and a second ship traveling south is $S(t)$ nautical miles south of the lighthouse at time $t$ (hours). The graphs of $W$ and $S$ are shown below. At what approximate rate is the distance between the ships increasing at $t = 1$? (nautical miles per hour = knots)




(A) 1 knot (B) 4 knots (C) 7 knots (D) 10 knots (E) 13 knots

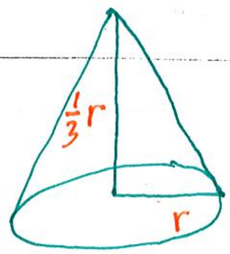
RR 6. Sand is being dumped on a pile in such a way that it always forms a cone whose base radius is always 3 times its height. The function $V$ whose graph is sketched in the figure gives the volume of the conical sand pile, $V(t)$, measured in cubic feet, after $t$ minutes. $\left(V(t) = \frac{1}{3}\pi r^2 h\right)$ At what approximate rate is the radius of the base changing after 6 minutes?




so $\frac{dV}{dt} = \frac{3\pi}{4}$

(A) 0.22 ft/min (B) 0.28 ft/min (C) 0.34 ft/min (D) 0.40 ft/min (E) 0.46 ft/min

Ans $\boxed{A}$




we have no information on $dh/dt$, so we need to rid h!

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi r^2 \left(\frac{1}{3}r\right)$

Eq: $\boxed{V = \frac{\pi}{9} r^3}$ $\longrightarrow$ Der: $\frac{dV}{dt} = \frac{\pi}{3} r^2 \frac{dr}{dt}$

$3\pi/4$    $\sqrt[3]{36}$    Find

$4\pi = \frac{\pi}{9} r^3$

$r = \sqrt[3]{36}$

$\frac{dr}{dt} = 0.206$ ft/min

Free Response

3. Oil is leaking from a pipeline on the surface of a lake and forms an oil slick whose volume increases at a constant rate of 2000 cubic centimeters per minute. The oil slick takes the form of a right circular cylinder with both its radius and height changing with time. (Note: The volume $V$ of a right circular cylinder with radius $r$ and height $h$ is given by $V = \pi r^2 h$.)

(a) At the instant when the radius of the oil slick is 100 centimeters and the height is 0.5 centimeter, the radius is increasing at the rate of 2.5 centimeters per minute. At this instant, what is the rate of change of the height of the oil slick with respect to time, in centimeters per minute? RR

(b) A recovery device arrives on the scene and begins removing oil. The rate at which oil is removed is $R(t) = 400\sqrt{t}$ cubic centimeters per minute, where $t$ is the time in minutes since the device began working. Oil continues to leak at the rate of 2000 cubic centimeters per minute. Find the time $t$ when the oil slick reaches its maximum volume. Justify your answer. OPT-ish

derivative function

derivative

(a) $V = \pi r^2 h$

$\frac{dV}{dt} = \pi\left(r^2 \frac{dh}{dt} + h\, 2r \frac{dr}{dt}\right)$

2000   100   Find   0.5   100   2.5

$\frac{dh}{dt} = 0.039$ cm/min

(b) $\frac{dV}{dt} = 2000 - R(t) = 0$

$R(t) = 2000$

$400\sqrt{t} = 2000$

$\sqrt{t} = 5$

$t = 25$

$\frac{dV}{dt}$ +   −   25

Max since $\frac{dV}{dt}$ changes from + to − at $t = 25$

RR 10. When the area of an expanding square, in square units, is increasing three times as fast as its side is increasing, in linear units, the side is

(A) $\frac{2}{3}$ (B) $\frac{3}{2}$ (C) 3 (D) 2 (E) 1

$A = s^2$

$3\,ds/dt$ → $\frac{dA}{dt} = 2s\frac{ds}{dt}$ ← Find

$3\cancel{ds/dt} = 2s\cancel{ds/dt}$

$3 = 2s$

$s = 3/2$

Ans $\boxed{B}$

11. What is the area of the largest rectangle that can be inscribed under the graph of $y = 2\cos x$ for $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$?

opt

(A) 2.20 units$^2$
(B) 2.24 units$^2$
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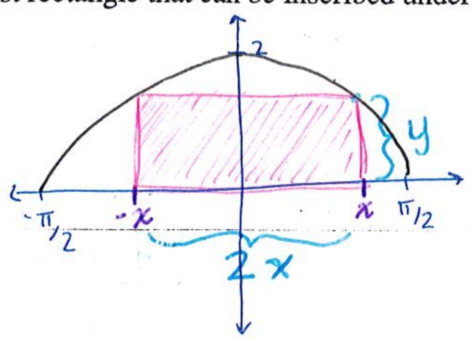


~~Area = x y~~

Area = length × width

$A = 2x(2\cos x) = 4x \cdot \cos x$

$A' = (\cos x)(4) + (-\sin x)(4x) = 0$

$A'$ − + − $x = \pm 0.860$ (plug back into Area equation)

-.86 .86

Ans $\boxed{B}$

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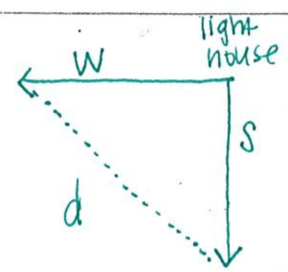
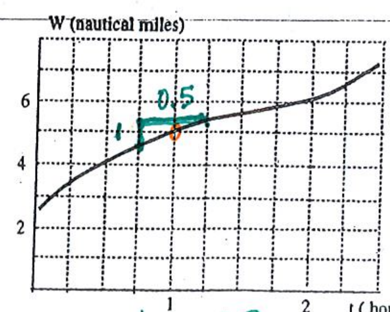
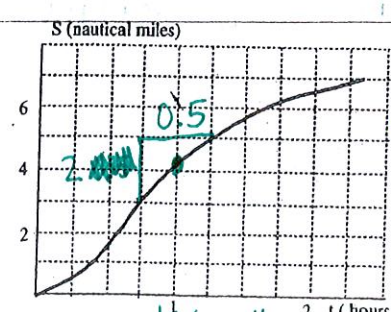



$dw/dt = 2$




$ds/dt = 4$

$W^2 + S^2 = d^2$

$\cancel{2}W\frac{dw}{dt} + \cancel{2}s\frac{ds}{dt} = \cancel{2}d\frac{dd}{dt}$

$5(2) + 4.2(4) = 6.5299\frac{dd}{dt}$

$\frac{dd}{dt} = 4.104$

(A) 1 knot (B) 4 knots (C) 7 knots (D) 10 knots (E) 13 knots

$(5)^2 + (4.2)^2 = d^2$

$d^2 = 42.64$

$d = 6.5299$

Ans $\boxed{B}$