$f'(x) = \frac{f(b)-f(a)}{b-a}$

Kuta Software - Infinite Calculus

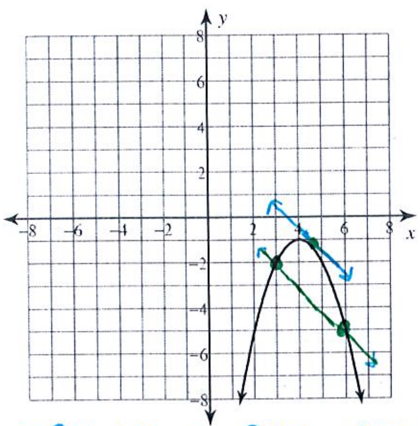
Name KEY

Mean Value Theorem

Date________________ Period____

For each problem, find the values of c that satisfy the Mean Value Theorem.

1) $y = -x^2 + 8x - 17$; $[3, 6]$



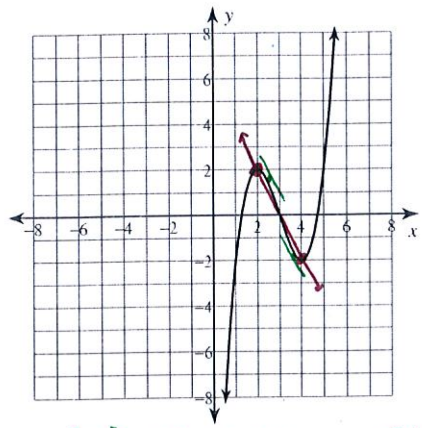
$-2x + 8 = \frac{f(6)-f(3)}{6-3}$

$-2x + 8 = \frac{-3}{3}$

$-2x = -9$

$\boxed{x = 9/2}$

2) $y = x^3 - 9x^2 + 24x - 18$; $[2, 4]$



$3x^2 - 18x + 24 = \frac{f(4)-f(2)}{4-2}$

$3x^2 - 18x + 24 = -2$

$3x^2 - 18x + 26 = 0$

quad Formula: $\boxed{x = 2.423, 3.577}$

3) $y = -\frac{x^2}{2} + x - \frac{1}{2}$; $[-2, 1]$

$-x + 1 = \frac{f(1)-f(-2)}{1--2}$

$-x + 1 = 1.5$

$-x = 0.5$

$\boxed{x = -0.5}$

4) $y = \frac{x^2}{2} - 2x - 1$; $[-1, 1]$

$x - 2 = \frac{f(1)-f(-1)}{1--1}$

$\boxed{x=0}$

$x - 2 = -2$

$x = 0$

5) $y = x^3 + 3x^2 - 2$; $[-2, 0]$

$3x^2 + 6x = \frac{f(0)-f(-2)}{0--2}$

$3x^2 + 6x = -2$

$3x^2 + 6x + 2 = 0$

quad. form: $\boxed{\begin{array}{l} x = -0.423 \\ x = -1.577 \end{array}}$

6) $y = -x^3 + 4x^2 - 3$; $[0, 4]$

$-3x^2 + 8x = \frac{f(4)-f(0)}{4-0}$

$\boxed{x = 8/3}$

$-3x^2 + 8x = 0$

$-x(3x - 8) = 0$

$x = 0, 8/3$

7) $y = \frac{x^2 - 9}{3x}$; $[1, 4]$

$\frac{(3x)(2x) - (x^2-9)(3)}{(3x)^2} = \frac{f(4)-f(1)}{4-1}$

$\frac{6x^2 - 3x^2 + 27}{9x^2} = \frac{13}{12}$

$\frac{3x^2 + 27}{9x^2} = \frac{13}{12}$

$117x^2 = 36x^2 + 324$ $\boxed{x = 2}$

$81x^2 - 324 = 0$

$81(x^2 - 4) = 0$

$81(x+2)(x-2) = 0$

8) $y = \frac{x^2}{2x - 4}$; $[-4, 1]$

$\frac{(2x-4)(2x) - (x^2)(2)}{(2x-4)^2} = \frac{f(1)-f(-4)}{1--4}$

$\frac{4x^2 - 8x - 2x^2}{(2x-4)^2} = \frac{1}{6}$

$\boxed{x = 2 - \sqrt{6}}$

$(2x-4)^2 = 12x^2 - 48x$

$4x^2 - 16x + 16 = 12x^2 - 48x$

$8x^2 - 32x - 16 = 0$

$8(x^2 - 4x - 2)$

9) $y = -(-2x+6)^{\frac{1}{2}}$; $[-2, 3]$

$-\frac{1}{2}(-2x+6)^{-1/2} \cdot -2 = \sqrt{4/10}$

$(-2x+6)^{-1/2} = 2/\sqrt{10}$

$\frac{1}{\sqrt{-2x+6}} = \frac{2}{\sqrt{10}}$

$\frac{1}{-2x+6} = \frac{4}{10}$

$10 = 4(-2x+6)$

$10 = -8x+24$

$-14 = -8x$

$x = 14/8$

$\boxed{x = 7/4}$

10) $y = -(-5x+25)^{\frac{1}{2}}$; $[3, 5]$

$-\frac{1}{2}(-5x+25)^{-1/2} \cdot -5 = \frac{\sqrt{5}}{\sqrt{2}}$

$\frac{5}{2\sqrt{-5x+25}} = \frac{\sqrt{5}}{\sqrt{2}}$

$\frac{25}{4(-5x+25)} = \frac{5}{2}$

$50 = 20(-5x+25)$

$\frac{5}{2} = -5x+25$

$-\frac{45}{2} = -5x$

$x = +\frac{45}{10} = 4.5$

$\boxed{x = 9/2}$

For each problem, determine if the Mean Value Theorem can be applied. If it can, find all values of *c* that satisfy the theorem. If it cannot, explain why not.

11) $y = -\frac{x^2}{4x+8}$; $[-3, -1]$

NO

note: ~~[illegible]~~ function undefined when $x = -2$, which is in interval

12) $y = \frac{-x^2+9}{4x}$; $[1, 3]$

YES

note: function undefined when $x=0$ but that's NOT in interval

$\frac{(4x)(-2x) - (-x^2+9)(4)}{16x^2} = -1$

$-16x^2 = -8x^2 + 4x^2 + 36$

$12x^2 - 36 = 0$

$12(x^2-3) = 0$

$x = \pm\sqrt{3}$

$\boxed{x = \sqrt{3}}$

13) $y = -(6x+24)^{\frac{2}{3}}$; $[-4, -1]$

YES

$\boxed{x = -28/9}$

14) $y = (x-3)^{\frac{2}{3}}$; $[1, 4]$

NO

note that $y' = \frac{2}{3}(x-3)^{-1/3} = \frac{2}{3} \cdot \frac{1}{(x-3)^{1/3}}$

so y is not differentiable at $x = 3$ which is in interval

Critical thinking question:

15) Use the Mean Value Theorem to prove that $|\sin a - \sin b| \le |a - b|$ for all real values of a and b where $a \ne b$.

no