Equations of Motion Review

1. The position of a particle is given by $s(t) = t^3 - 6t^2 + 9t$ where t is measured in seconds and s in meters.

a. Find the velocity at time t. $v(t) = 3t^2 - 12t + 9$

b. What is the velocity after 2 s? After 4s? $v(2) = -3$ m/s + $v(4) = 9$ m/s

c. When is the particle at rest?

$v(t) = 3t^2 - 12t + 9 = 0$

$3(t^2 - 4t + 3) = 0$

$(t-3)(t-1) = 0$ → $t = 1, 3$ sec

$v(t)$: 0 + 1 − 3 +

d. When is the particle moving right? $(0,1) \cup (3,\infty)$ sec
(when $v > 0$)

e. When is the particle moving left? $(1,3)$ sec
(when $v < 0$)

f. Find the acceleration at time t. $a(t) = 6t - 12$

g. Find the acceleration at t=4s. $a(4) = 12\ m/s^2$

$a(t) = 6t - 12 = 0$

$t = 2$

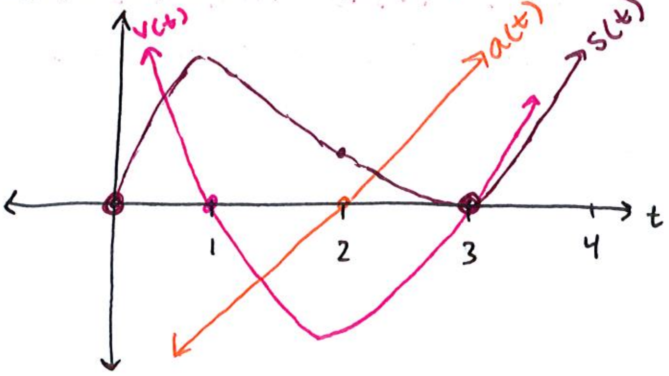
$a(t)$: 0 − 2 +

h. When is the velocity increasing? $(2,\infty)$
(when $a(t) > 0$)

i. When is the particle speeding up? $(1,2) \cup (3,\infty)$ sec
(when a + v have same signs)

j. When is the particle slowing down? $(0,1) \cup (2,3)$ sec
(when a + v have opp. signs)

k. Sketch a graph with position, velocity, and acceleration on the same graph.



X-int: $0 = t^3 - 6t^2 + 9t$

$0 = t(t^2 - 6t + 9)$

$0 = t(t-3)^2$

$t = 0, 3$

y-int: $s(0) = 0$

2. A particle moves along a line with its position given by $s(t) = te^{-t/2}$ where t is measured in second and s in feet.

a. Find the velocity at time t. $v(t) = e^{-t/2} \cdot 1 + t \cdot -\frac{1}{2}e^{-t/2} = e^{-t/2} - \frac{t}{2}e^{-t/2} = v(t)$

b. What is the velocity after 2 s? After 4s? $v(2) = 0$ ft/sec + $v(4) = -e^{-2}$ ft/sec

c. When is the particle at rest?

$v(t) = \underbrace{e^{-t/2}}_{t = \text{none}}\underbrace{(1 - \frac{1}{2}t)}_{t=2} = 0$

$v(t)$: 0 + 2 −

d. When is the particle moving right? $(0,2)$ sec

e. When is the particle moving left? $(2,\infty)$ sec

f. Find the acceleration at time t. $a(t) = -\frac{1}{2}e^{-t/2} - (e^{-t/2} \cdot + \frac{1}{2} + \frac{t}{2} \cdot -\frac{1}{2}e^{-t/2}) = \frac{t}{4}e^{-t/2} - e^{-t/2}$

$= e^{-t/2}(\frac{t}{4} - 1) = a(t)$

g. Find the acceleration at t=4s. $a(4) = 0$ m/sec

$a(t) = \underbrace{(\frac{t}{4} - 1)}_{t=4}\underbrace{(e^{-t/2})}_{t=\text{none}} = 0$

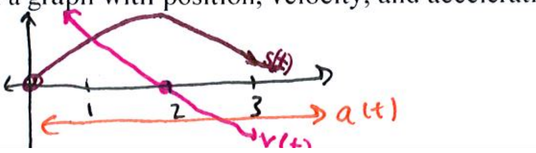
$a(t)$: 0 − 4 +

h. When is the velocity increasing? $(4,\infty)$ sec

i. When is the particle speeding up? $(2,4)$ sec

j. When is the particle slowing down? $(0,2) \cup (4,\infty)$

k. Sketch a graph with position, velocity, and acceleration on the same graph.

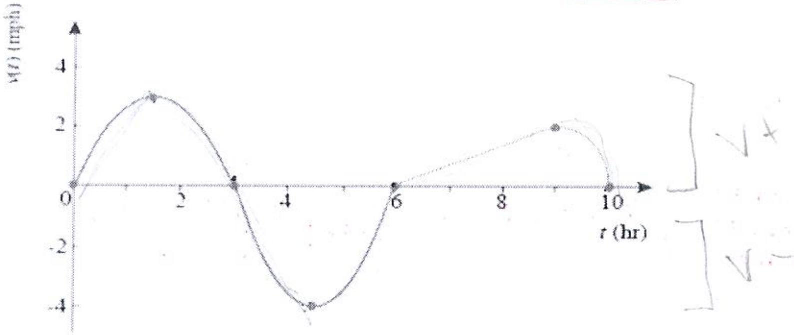


x-int: $0 = te^{-t/2}$

$t = 0$

y-int: $s(0) = 0$

3. Brooke is strolling through the forest. Her $\boxed{\text{velocity}}$ East is given by the following graph:



(a) At what time(s) is Brooke not moving? t = 0, 3, 6, 10 hrs
(when v(t) = 0)

(b) On what time interval(s) is Brooke moving East? (0,3) ∪ (6,10) hrs
(East → Positive direction... when v > 0)

(c) On what time interval(s) is Brooke moving West? (3,6) hrs
(West → Negative direction... when v < 0)

(d) At what time(s) is Brooke moving most rapidly East? t ≈ 1.5 hr
(when vel is greatest above x-axis)

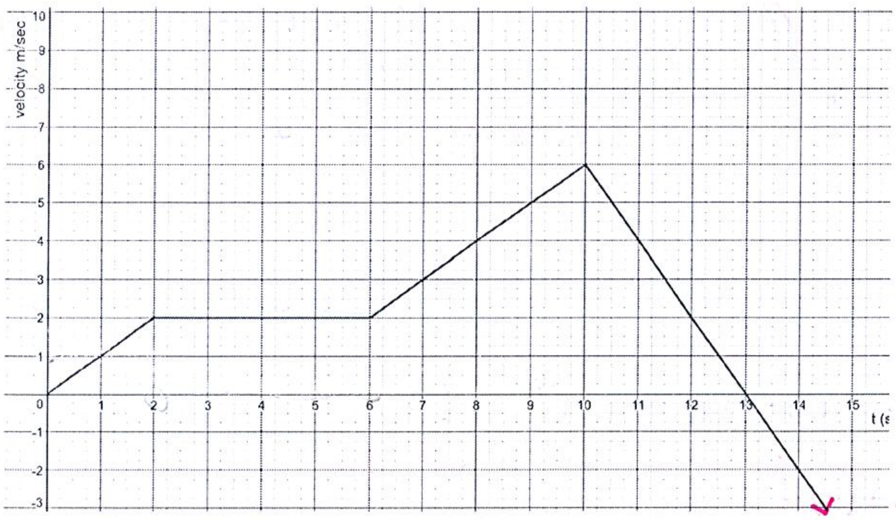
(e) At what time(s) is Brooke moving most rapidly West? t ≈ 4
(when vel is smallest below x-axis)

(f) At what time(s) is Brooke moving most rapidly? t ≈ 4 hr
(Speed is greatest → |v(t)|)

(g) On what time interval(s) is Brooke speeding up? (0,1) ∪ (3,4) ∪ (6,9) hrs
(when v + a have same signs)

(h) On what time interval(s) is Brooke slowing down? (1,3) ∪ (9,10) ∪ (4,6) hrs
(when v + a have opposite signs)

4.



Using the above graph of velocity, answer the following…

a. When is the particle moving left? (13, ∞) sec

b. When is the particle moving right? (0,13) sec

c. When is the particle the furthest right? t = 13

d. When is the acceleration positive? (0,2) ∪ (6,10) sec

e. When is the acceleration negative? (10, ∞) sec

f. When is the acceleration 0? (2,6) sec

g. When is the particle at rest? t = 0, 13 sec

H. When speeding up? (0,2) ∪ (6,10) ∪ (13,∞)

I. When slowing down? (10,13) sec

J. What is acceleration at t=8?
$a(8) = 1 \text{ m/sec}^2$

oordinates s of a moving body for various values of t are given. Assume position is a differentiable unction.

1.75 ↓ 2.75 ↓

t(seconds)	0	.5	1	1.5	2	2.5	3	3.5	4
s (ft)	12.5	26	36.5	44	48.5	50	48.5	44	36.5

Estimate the velocity at t=1.75 seconds. $v(1.75) \approx \frac{s(2)-s(1.5)}{2-1.5} = \frac{48.5-44}{0.5} = \frac{4.5}{0.5} = 9$ ft/sec

Estimate the velocity at t=2.75 seconds. $v(2.75) \approx \frac{v(3)-v(2.5)}{3-2.5} = \frac{48.5-50}{0.5} = \frac{-1.5}{0.5} = 3$ ft/sec

Is the particle ever at rest? How do you know?

yes → "rest" means $v(t)=0$ ... OR $s'(t)=0$.
Because $s(t)$ appears to be increasing + decreasing ($s'>0$ + $s'<0$) then by IVT, there had to be some t value(s) such that $s'(t)=0$ ... $v(t)=0$.

6. Given this table of velocity as a function of time, answer the following questions. Assume velocity is a differentiable function.

0.5 ↓ 3.5 ↓

t(seconds)	0	1	2	3	4	5
v(m/s)	0	-2	0	1	3	2

Estimate the acceleration at t=0.5. $a(0.5) \approx \frac{v(1)-v(0)}{1-0} = \frac{-2-0}{1} = -2$ m/s²

Estimate the acceleration at t=3.5. $a(3.5) \approx \frac{v(4)-v(3.5)}{4-3} = \frac{3-1}{1} = \frac{2}{0.5} = 4$ m/s²

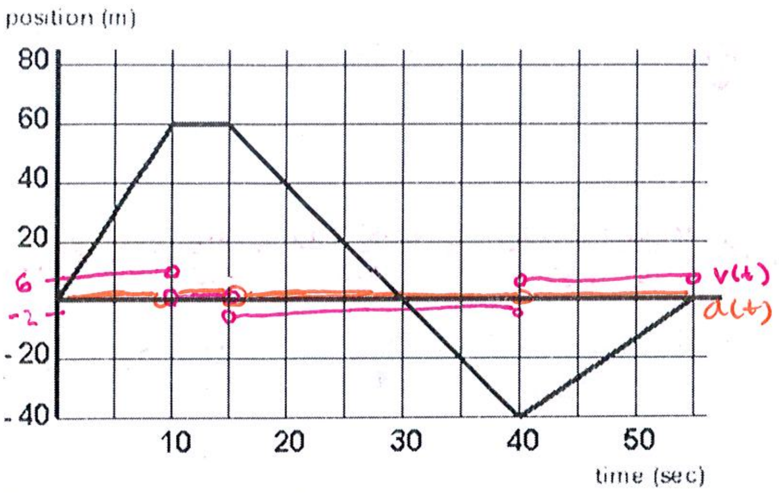
Is the particle ever moving left? yes because $v<0$ at some values of t

Is the particle ever moving right? yes because $v>0$ at some values of t

Is the acceleration ever 0?

yes → $a(t)=0$ happens when v' changes signs.
Because $v(t)$ appears to be increasing + decreasing ($v'>0$ + $v'<0$) then by IVT, there had to be some t value(s) such that $v'(t)=0$ ... $a(t)=0$.

7.



Given this position graph…

a. Find the velocity at time t=12 v(12) = 0 m/sec

b. What is the velocity after 5 s? After 0s? v(5) = 6 m/sec + v(10) = DNE

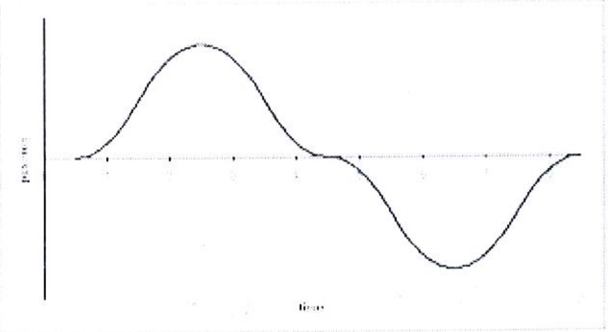
c. When is the particle at rest? (10, 15) sec

d. When is the particle moving right? (0, 10) ∪ (40, 55) sec

e. When is the particle moving left? (15, 40) sec

k. Sketch a graph with position, velocity, and acceleration on the same graph.

8.



The above graph is position (m) vs time (seconds).

a. When is the velocity positive? (0, 2.5) ∪ (6.5, 8)

b. When is the velocity negative? (2.5, 6.5)

c. When does the particle change directions? t = 2.5, 6.5

d. When is the particle at rest? t = 0, 2.5, 6.5

~~e. When is the acceleration positive~~?

~~f. When is the acceleration negative?~~

~~g. When is the particle speeding up~~?

~~h. When is the particle slowing down~~?