

P. 427 #1-4, 11-12

$$\begin{aligned} 1. \quad \int_0^4 (5x - x^2) - (x) \, dx &= \int_0^4 4x - x^2 \, dx \\ &= \left[\frac{4x^2}{2} - \frac{1x^3}{3} \right]_0^4 \\ &= 2(4)^2 - \frac{(4)^3}{3} - 0 \\ &= 32 - \frac{64}{3} = \boxed{\frac{32}{3}} \end{aligned}$$

$$\begin{aligned} 2. \quad \int_0^2 (\sqrt{x+2}) - \left(\frac{1}{x+1}\right) \, dx \\ &= \left[\frac{2(x+2)^{3/2}}{3} - \ln|x+1| \right]_0^2 \\ &= \left(\frac{2(2+2)^{3/2}}{3} - \ln|2+1| \right) - \left(\frac{2(0+2)^{3/2}}{3} - \ln|0+1| \right) \\ &= \frac{16}{3} - \ln 3 - \frac{2\sqrt{8}}{3} + \cancel{\ln 1} \\ &= \frac{16 - 4\sqrt{2}}{3} - \ln 3 = \boxed{\frac{4}{3}(4 - \sqrt{2}) - \ln 3} \end{aligned}$$

$$\begin{aligned}
3. \quad \int_{-1}^1 (e^y) - (y^2 - 2) \, dy &= \int_{-1}^1 (e^y - y^2 + 2) \, dy \\
&= \left[e^y - \frac{1}{3}y^3 + 2y \right]_{-1}^1 \\
&= (e^1 - \frac{1}{3}(1)^3 + 2(1)) - (e^{-1} - \frac{1}{3}(-1)^3 + 2(-1)) \\
&= (e - \frac{1}{3} + 2) - (\frac{1}{e} + \frac{1}{3} - 2) \\
&= e - \frac{1}{3} + 2 - \frac{1}{e} - \frac{1}{3} + 2 = e - \frac{1}{e} - \frac{2}{3} + 4 = \boxed{e - \frac{1}{e} + \frac{10}{3}}
\end{aligned}$$

$$\begin{aligned}
4. \quad \int_0^3 (2y - y^2) - (y^2 - 4y) \, dy &= \int_0^3 6y - 2y^2 \, dy \\
&= \left[\frac{6y^2}{2} - \frac{2y^3}{3} \right]_0^3 \\
&= \left[3y^2 - \frac{2}{3}y^3 \right]_0^3 \\
&= (3(3)^2 - \frac{2}{3}(3)^3) - 0 \\
&= 27 - 18 \\
&= \boxed{9}
\end{aligned}$$

$$11. \quad x = 1 - y^2, \quad x = y^2 - 1$$

find bounds: $1 - y^2 = y^2 - 1$

$$2y^2 - 2 = 0$$

$$2(y^2 - 1) = 0$$

$$2(y+1)(y-1) = 0$$

$$\boxed{y = \pm 1} \text{ BOUNDS}$$

plug in ~~1~~ $y=0$ to find "right" function

$$x = 1 - 0^2 = 1 \leftarrow \text{right}$$

$$x = 0^2 - 1 = -1 \leftarrow \text{left}$$

$$\int_{-1}^1 (1 - y^2) - (y^2 - 1) dy = \int_{-1}^1 -2y^2 + 2 dy$$

$$= \left[-\frac{2y^3}{3} + 2y \right]_{-1}^1$$

$$= \left(-\frac{2(1)^3}{3} + 2(1) \right) - \left(-\frac{2(-1)^3}{3} + 2(-1) \right)$$

$$= -\frac{2}{3} + 2 - \frac{2}{3} + 2$$

$$= 4 - \frac{4}{3} = \boxed{\frac{8}{3}}$$

$$12. \quad \underbrace{4x + y^2 = 12}_{\downarrow}, \quad x = y$$

$$x = \frac{12 - y^2}{4}$$

$$x = 3 - \frac{1}{4}y^2$$

find bounds: $3 - \frac{1}{4}y^2 = y$

$$12 - y^2 = 4y$$

$$y^2 + 4y - 12 = 0$$

$$(y + 6)(y - 2) = 0$$

$$\boxed{y = -6, 2} \text{ BOUNDS}$$

plug in $y = 0$ to find "right" function

$$x = 3 - \frac{1}{4}(0)^2 = 3 \leftarrow \text{right}$$

$$x = 0 \leftarrow \text{left}$$

$$\int_{-6}^2 (3 - \frac{1}{4}y^2) - (y) dy = \left[3y - \frac{1}{4} \cdot \frac{y^3}{3} - \frac{y^2}{2} \right]_{-6}^2$$

$$= \left(3(2) - \frac{(2)^3}{12} - \frac{(2)^2}{2} \right) - \left(3(-6) - \frac{(-6)^3}{12} - \frac{(-6)^2}{2} \right)$$

$$= 6 - \frac{2}{3} - 2 + 18 - \frac{216}{12} + \frac{36}{2}$$

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