Optimization Worksheet Name_______________________________

Solve each problem.

1. A cylindrical can is to be made to hold 1000 mL of oil. Find the dimensions that will minimize the surface area of this can.

$SA = 2\pi r^2 + 2\pi r h$

$= 2\pi r^2 + 2\pi r\left(\frac{1000}{\pi r^2}\right)$

$= 2\pi r^2 + \frac{2000}{r}$

$= 2\pi r^2 + 2000r^{-1}$

$SA' = 4\pi r - 2000r^{-2} = 0/DNE$

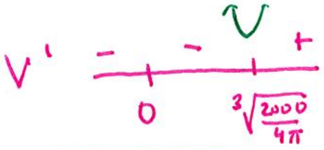
$4\pi r = \frac{2000}{r^2}$

$4\pi r^3 = 2000$

$r = \sqrt[3]{2000/4\pi}\ ,0$

$V = \pi r^2 h = 1000$

$h = \frac{1000}{\pi r^2}$



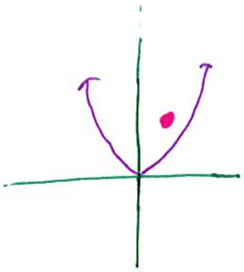
dimensions:

$r = \sqrt[3]{500/\pi} = 5.419$ cm

$h = 1000/\pi(\sqrt[3]{500/\pi})^2 = 10.839$ cm

2. Find the point(s) on the parabola $y = x^2$ that is closest to the point (1,4)

points: $(\overset{x_1}{1}, \overset{y_1}{4})$ $(\overset{x_2}{x}, \overset{y_2}{x^2})$



$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(x-1)^2 + (x^2-4)^2}$

$= \sqrt{x^2 - 2x + 1 + x^4 - 8x^2 + 16}$

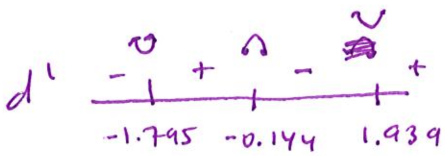
$= \sqrt{x^4 - 9x^2 + 2x + 17}$

$d' = \frac{1}{2}(x^4 - 7x^2 - 2x + 17)^{-1/2} \cdot (4x^3 - 14x - 2) = 0/DNE$

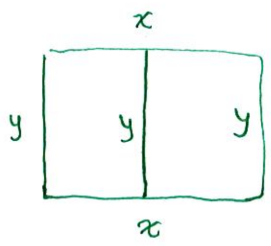
$= \frac{4x^3 - 14x - 2}{2\sqrt{x^4 - 7x^2 - 2x + 17}}$

$x = -1.795$

$x = 1.939$



3. A farmer wants to fence an area of 1.5 million square feet in a rectangular field and then divide it in half with a fence parallel to one of the sides of the rectangle. How can he do this so as to minimize the cost of the fence?



$SA = 2x + 3y$

$= 2x + 3\left(\frac{1500{,}000}{x}\right)$

$= 2x + 4{,}500{,}000x^{-1}$

$SA' = 2 - 4{,}500{,}000x^{-2} = 0/DNE$

$2 - \frac{4{,}500{,}000}{x^2} = 0$

$2 = \frac{4{,}500{,}000}{x^2}$

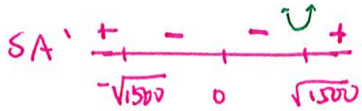
$2x^2 = 4{,}500{,}000$

$x^2 = 2{,}250{,}000$

$x = \pm\sqrt{2{,}250{,}000} = \pm\sqrt{1500}, 0$

$A = xy = 1{,}500{,}000$

$y = \frac{1{,}500{,}000}{x}$



dimensions:

$\sqrt{1500} \times \frac{1{,}500{,}000}{\sqrt{1500}}$

feet

1500 x 1000

ft

4. A model for the US average price of a pound of white sugar from 1993 to 2003 is given by the function

$$S(t) = -0.00003237t^5 + 0.0009037t^4 - 0,008956t^3 + 0.03629t^2 - 0.04458t + .4074$$

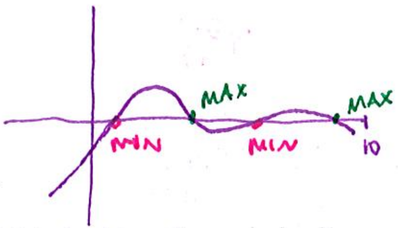
where t is measured in years since August of 1993. Find the times when sugar was the cheapest and most expensive during the period 1992-2003. ← closed interval

$S'(t) = 5(-0.00003237)t^4 + 4(0.0009037)t^3 - 3(0.008956)t^2 + 2(0.03629)t - 0.04458$

$= 0/DNE$

to see graph: zoom in on y-values

[x-window: [0, 10] , y-window: [-0.1, 0.1]]



min; $x = 0.855$ } plug into S(t)
$x = 7.292$ } to see abs. min
$x = 10$

max: $x = 4.618$ } plug into S(t)
$x = 9.5699$ } to see abs. max
$x = 0$

Cheapest: t=.855
expensive: x=4.618

5. Oil is leaking from a pipeline on the surface of a lake and forms an oil slick whose volume increases at a constant rate of 2000 cubic centimeters per minute. ~~The oil slick takes the form of a right circular cylinder~~

A recovery device arrives on the scene and begins removing oil. The rate at which oil is removed is $R(t) = 400\sqrt{t}$ cubic centimeters per minute, where t is the time in minutes since the device began working. Oil continues to leak at the rate of 2000 cubic centimeters per minute. Find the time t when the oil slick reaches its maximum volume. Justify your answer.

Leaking Rate: 2000

Recovery Rate: $400\sqrt{t}$

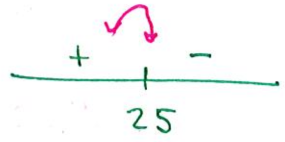
$F' = 2000 - 400\sqrt{t} = 0/DNE$

$2000 = 400\sqrt{t}$

$5 = \sqrt{t}$

$t = 25$ min

F'



Oil slick reaches max at t = 25 min since that's when F' changes from + to −