p. 427 #5-10 Area wrt x

5. $y = e^x$, $y = x^2 - 1$, $\boxed{x = -1, \; x = 1}$

BOUNDS

plug in $x = 0$ to find "top" function

$y = e^0 = 1$ ← top

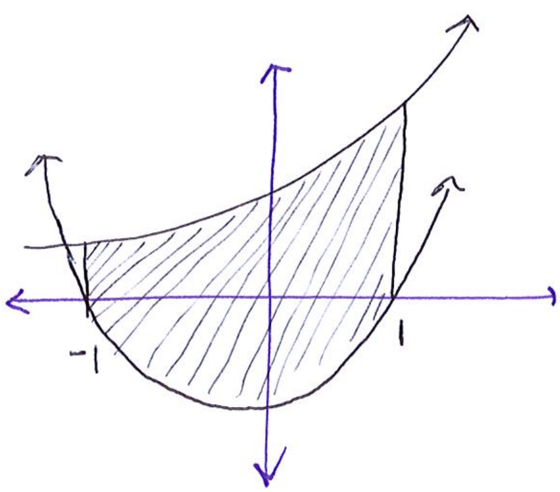
$y = 0^2 - 1 = -1$ ← bottom

$$\boxed{\int_{-1}^{1} (e^x - x^2 + 1) \, dx}$$ ← Definite Integral

$$= e^x - \tfrac{1}{3}x^3 + x \Big]_{-1}^{1}$$

$$= (e^1 - \tfrac{1}{3} + 1) - (e^{-1} + \tfrac{1}{3} - 1)$$

$$= e - \tfrac{1}{3} + 1 - \tfrac{1}{e} - \tfrac{1}{3} + 1 = \boxed{e - \tfrac{1}{e} + \tfrac{4}{3}}$$ ← area



6. $y = \sin x$, $y = x$, $\boxed{x = \pi/2, \quad x = \pi}$

BOUNDS

plug in $x = 3\pi/4$ to find "top" function

$y = \sin 3\pi/4 = \sqrt{2}/2$ ← BOTTOM

$y = 3\pi/4 = 3\pi/4$ ← TOP

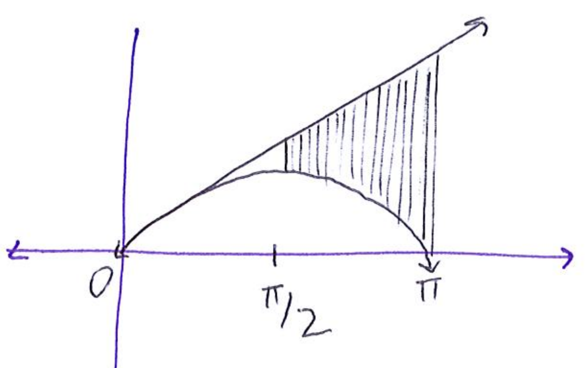
$$\boxed{\int_{\pi/2}^{\pi} (x - \sin x)\, dx}$$ ← Definite Integral

$$= \frac{1}{2}x^2 + \cos x \Big]_{\pi/2}^{\pi}$$

$$= \left(\frac{1}{2}(\pi)^2 + \cos \pi\right) - \left(\frac{1}{2}(\pi/2)^2 + \cos \pi/2\right)$$

$$= \left(\frac{\pi^2}{2} - 1\right) - \left(\frac{\pi^2}{8} + 0\right)$$

$$= \boxed{\frac{3\pi^2}{8} - 1}$$ ← area



7. $y = (x-2)^2 \qquad y = x$

find bounds:

$$(x-2)^2 = x$$
$$x^2 - 4x + 4 = x$$
$$x^2 - 5x + 4 = 0$$
$$(x-4)(x-1) = 0$$
$$\boxed{x = 1, 4} \text{ BOUNDS}$$

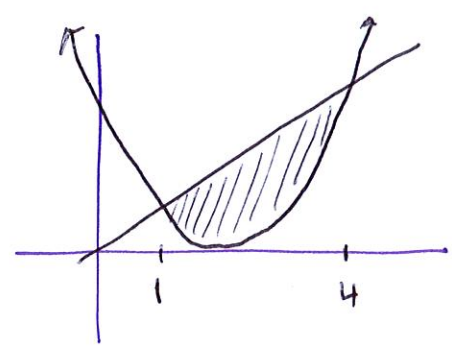
plug in $x = 2$ to find "top" function

$y = (2-2)^2 = 0$ ← bottom

$y = 2 = 2$ ← top

$$\boxed{\int_1^4 \left[x - (x-2)^2\right] dx}$$ ← definite integral

$$= \int_1^4 -x^2 + 5x - 4 \, dx$$



$$= -\frac{1}{3}x^3 + \frac{5x^2}{2} - 4x \Big]_1^4$$

$$= \left(-\frac{1}{3}(4)^3 + \frac{5(4)^2}{2} - 4(4)\right) - \left(-\frac{1}{3} + \frac{5}{2} - 4\right)$$

$$= \boxed{9/2}$$ ← area

8. $y = x^2 - 2x, \quad y = x + 4$

find bounds: $x^2 - 2x = x + 4$

$x^2 - 3x - 4 = 0$

$(x-4)(x+1) = 0$

$\boxed{x = -1, 4}$ BOUNDS

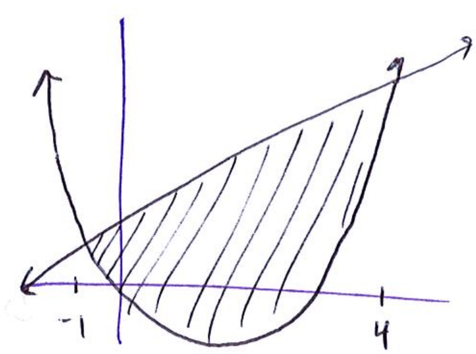
plug in $x=0$ to find function on "top"

$y = 0^2 - 2(0) = 0 \leftarrow$ bottom

$y = 0 + 4 = 4 \leftarrow$ top

$$\boxed{\int_{-1}^{4} [x + 4 - (x^2 - 2x)]\,dx} \leftarrow \text{Definite Integral}$$

$$\int_{-1}^{4} (-x^2 + 3x + 4)\,dx$$



$$= -\frac{1}{3}x^3 + \frac{3x^2}{2} + 4x \Big]_{-1}^{4}$$

$$= \left[-\frac{1}{3}(4)^3 + \frac{3(4)^2}{2} + 4(4)\right] - \left[-\frac{1}{3}(-1)^3 + \frac{3(-1)^2}{2} + 4(-1)\right]$$

$$= \left(-\frac{64}{3} + 24 + 16\right) - \left(\frac{1}{3} + \frac{3}{2} - 4\right)$$

$$= \boxed{\frac{125}{6}} \leftarrow \text{area}$$

9. $y = \frac{1}{x}$, $\quad y = \frac{1}{x^2}$, $\quad \boxed{x = 2}$ ONE BOUND

find other bound: $\quad \frac{1}{x} = \frac{1}{x^2}$

$$x^2 = x$$

$$x^2 - x = 0$$

$$x(x-1) = 0$$

$$\boxed{x = \underset{\text{no!}}{\cancel{0}},\ 1,\ 2} \text{ BOUNDS}$$

note: $x=0$ is NOT a bound

plug in $x = 3/2$ to find "top" function:

$$y = \frac{1}{(3/2)} = 2/3 \leftarrow \text{top}$$

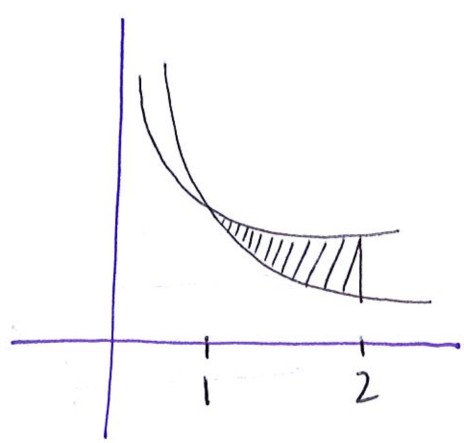
$$y = \frac{1}{(3/2)^2} = 4/9 \leftarrow \text{bottom}$$

$$\boxed{\int_1^2 \left(\frac{1}{x} - \frac{1}{x^2}\right) dx} \leftarrow \text{definite integral}$$

$$= \ + \frac{1}{x} \Big]_1^2$$

$$= \left(\ln|2| + \frac{1}{2}\right) - \left(\ln|1| + 1\right)$$

$$= \boxed{\ln 2 - \frac{1}{2}} \leftarrow \text{area}$$



10. $y = \sin x, \quad y = \frac{2x}{\pi}, \quad x \geq 0$

find bound: $\sin x = \frac{2x}{\pi}$ } yikes! hard to solve algebraically. solve analytically. (think about it)

$\boxed{x = 0, \pi/2}$

BOUNDS

plug in $x = \pi/4$, to find "top" function

$y = \sin \pi/4 = \sqrt{2}/2 \leftarrow$ top

$y = \frac{2(\pi/4)}{\pi} = \frac{\pi/2}{\pi} = \frac{1}{2} \leftarrow$ bottom

$$\boxed{\int_0^{\pi/2} \left(\sin x - \frac{2x}{\pi}\right) dx} \leftarrow \text{definite integral}$$

$$= -\cos x - \frac{\frac{2}{\pi} x^2}{2} \Big]_0^{\pi/2}$$

$$= -\cos x - \frac{1}{\pi} x^2 \Big]_0^{\pi/2}$$

$$= \left(-\cos \pi/2 - \pi/4\right) - \left(-\cos 0 - \frac{1}{\pi} \cdot 0^2\right)$$

$$= 0 - \pi/4 - (-1 - 0)$$

$$= \boxed{1 - \pi/4} \leftarrow \text{area}$$