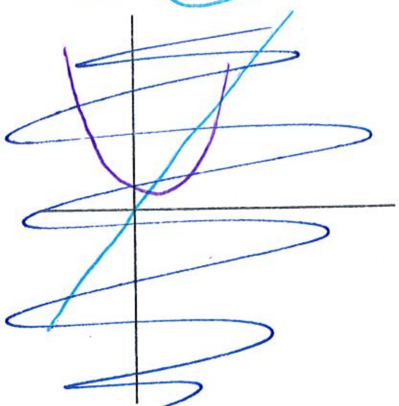
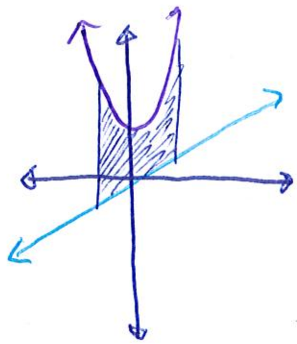
AP Calculus AB Name________________________

Area Between Two Curves Worksheet HW

Sketch the graphs, shade the bounded region and find the area bounded by the given expressions. **(NO CALCULATORS FOR PROBLEMS #1-9)**

1) $y = x^2 + 1$, $y = x$, $x = -1$, and $x = 2$





$$\boxed{\int_{-1}^{2} (x^2+1)-(x)\ dx}$$

$$= \frac{1}{3}x^3 + x - \frac{1}{2}x^2 \Big]_{-1}^{2}$$

$$= \left[\frac{1}{3}(2)^3 + (2) - \frac{1}{2}(2)^2\right] - \left[\frac{1}{3}(-1)^3 + (-1) - \frac{1}{2}(-1)^2\right]$$

$$= \left[\frac{8}{3} + 2 - 2\right] - \left[-\frac{1}{3} - 1 - \frac{1}{2}\right] = 4.5$$

2) $y = \sqrt{x}$ and $y = \dfrac{x}{4}$

BOUNDS:

$\sqrt{x} = \frac{x}{4}$

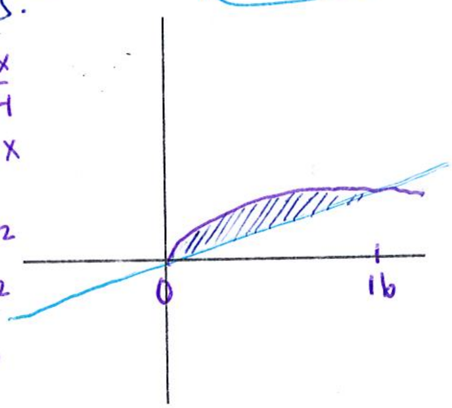
$4\sqrt{x} = x$

$4 = \frac{x}{x^{1/2}}$

$4 = x^{1/2}$

$x = 16$

$x = 0$



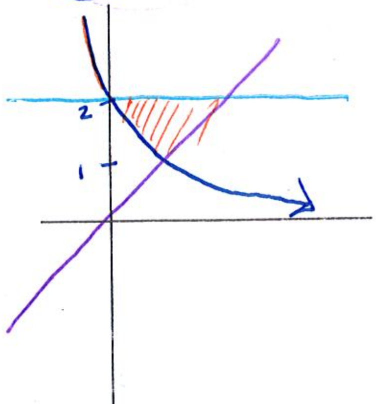
$$\boxed{\int_{0}^{16} \sqrt{x} - \frac{x}{4}\ dx}$$

$$= \frac{2}{3}x^{3/2} - \frac{1}{8}x^2 \Big]_{0}^{16}$$

$$= \left[\frac{2}{3}(16)^{3/2} - \frac{1}{8}(16)^2\right] - [0]$$

$$= \frac{128}{3} - 32 = \frac{32}{3}$$

3) $x = \dfrac{1}{y^2}$, $y = x$ and $y = 2$



$$\boxed{\int_{1}^{2} (y) - \left(\frac{1}{y^2}\right)\ dy}$$

$$\frac{1}{2}y^2 + y^{-1} \Big]_{1}^{2}$$

$$= \left[\frac{1}{2}(2)^2 + (2)^{-1}\right] - \left[\frac{1}{2}(1)^2 + (1)^{-1}\right]$$

$$= \left[2 + \frac{1}{2}\right] - \left[\frac{1}{2} + 1\right] = 1$$

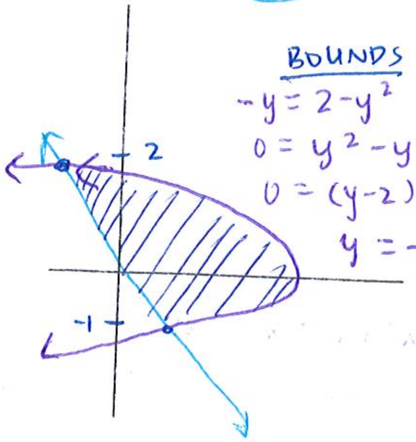
BOUNDS

$\frac{1}{y^2} = y$

$1 = y^3$

$y = 1$

4) $x = 2 - y^2$, $y = -x$, and $\cancel{y=0}$



BOUNDS

$-y = 2 - y^2$

$0 = y^2 - y - 2$

$0 = (y-2)(y+1)$

$y = -1, 2$

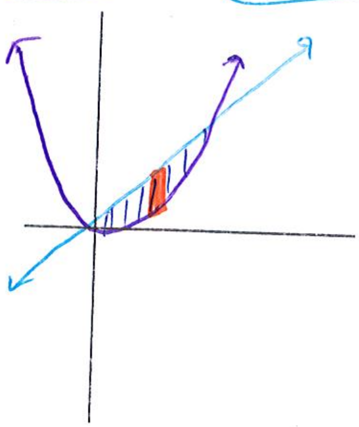
$$\boxed{\int_{-1}^{2} (2-y^2) - (-y) \quad dx}$$

$= 2y - \frac{1}{3}y^3 + \frac{1}{2}y^2 \Big]_{-1}^{2}$

$= \left[2(2) - \frac{1}{3}(2)^3 + \frac{1}{2}(2)^2\right] - \left[2(-1) - \frac{1}{3}(-1)^3 + \frac{1}{2}(-1)^2\right]$

$= \left[4 - \frac{8}{3} + 2\right] - \left[-2 + \frac{1}{3} + \frac{1}{2}\right] = 4.5$

5) $y = x^2$ and $y = 4x$ ** use *vertical* rectangles (***dx***)



BOUNDS:

$4x = x^2$

$0 = x^2 - 4x$

$0 = x(x-4)$

$x = 0, 4$

$$\boxed{\int_0^4 4x - x^2 \quad dx}$$

$= 2x^2 - \frac{1}{3}x^3 \Big]_0^4$

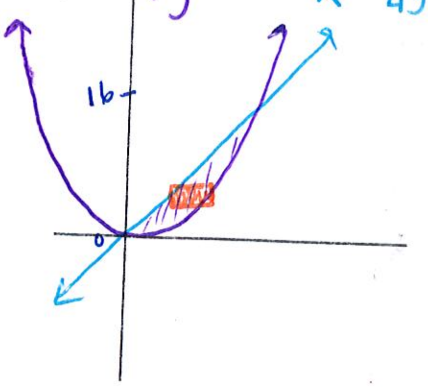
$= \left[2(4)^2 - \frac{1}{3}(4)^3\right] - [0]$

$= 32 - \frac{64}{3} = \boxed{\frac{32}{3}}$

6) $y = x^2$ and $y = 4x$ ** use *horizontal* rectangles (***dy***)

* notice #5 = #6

$x = \sqrt{y}$ $x = \frac{1}{4}y$



BOUNDS:

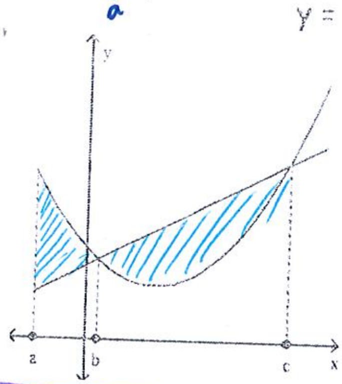
$y = 0, 16$

$$\int_0^{16} \sqrt{y} - \frac{1}{4}y \, dy = \frac{2}{3}y^{3/2} - \frac{1}{8}y^2 \Big]_0^{16}$$

$= \left[\frac{2}{3}(16)^{3/2} - \frac{1}{8}(16)^2\right] - [0]$

$= 32/3$

7. Set up definite integrals that represent the indicated shaded areas over the interval $[a, c]$. $\int_a^b g(x) - f(x)\,dx + \int_b^c f(x) - g(x)\,dx$



#8

BOUNDS:

$x^3 - x = 3x$

$x^3 - 4x = 0$

$x(x^2 - 4) = 0$

$x(x+2)(x-2) = 0$

$x = 0, \pm 2$

8. Find the area bounded between the functions $y = x^3 - x$ & $y = 3x$

$$\int_0^2 (3x) - (x^3 - x)\,dx + \int_{-2}^0 (x^3 - x) - (3x)\,dx$$

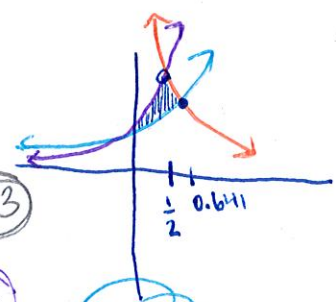
9. Find the area bounded between the functions $y = x^2, y = \frac{1}{x}$ & $y = 4$

$$\int_1^4 \sqrt{y} - \frac{1}{y}\,dy = \frac{2}{3}y^{3/2} - \ln|y|\Big]_1^4 = \left(\frac{2}{3}(4)^{3/2} - \ln 4\right) - \left(\frac{2}{3}\right) = \boxed{\frac{14}{3} - \ln 4}$$

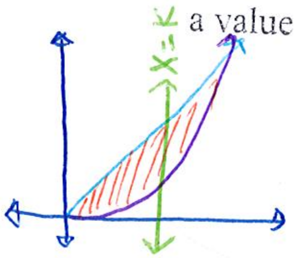
*** graphing calculators are required for problems #10 – 12:*

10. Find the area bounded by $y = 2^x$, $y = 4^x$ and $y = \frac{1}{x}$.

$$\int_0^{1/2} (4^x - 2^x)\,dx + \int_{1/2}^{.641} \left(\frac{1}{x} - 2^x\right)dx = .163$$



11. Consider the region in Quadrant I bounded by the functions $y = x^3$ and $y = 4x$. Find a value of k so the line $x = k$ divides the region into two regions of equal area.



$$\int_0^k 4x - x^3\,dx = \int_k^2 4x - x^3\,dx$$

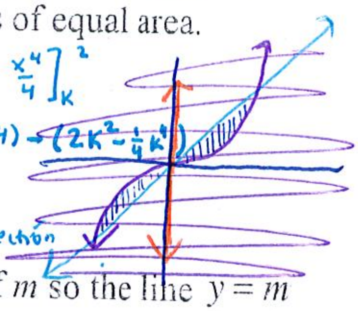
$$= \frac{4x^2}{2} - \frac{x^4}{4}\Big]_0^k = \frac{4x^2}{2} - \frac{x^4}{4}\Big]_k^2$$

$$= \left(2k^2 - \frac{1}{4}k^4\right) - (0) = (8-4) - \left(2k^2 - \frac{1}{4}k^4\right)$$

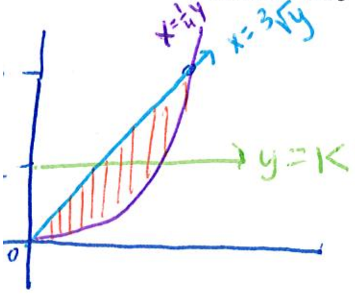
$$= 4k^2 + \frac{1}{2}k^4 = 4$$

$K = 1.082$

graph + find intersection in calculator



12. Consider the same region from problem #11; now find a value of m so the line $y = m$ divides the region into two regions of equal area.



$$\int_0^K \left(\frac{1}{4}y - \sqrt[3]{y}\right)dy = \int_K^8 \left(\frac{1}{4}y - \sqrt[3]{y}\right)dy$$

switch

oops!

switch

oops!

$K = 2.829$

Find x

① $\int_2^x \frac{6}{t}\,dt = 9$

$= 6\ln|t|\Big]_2^x = 9$

$= 6\ln|x| - 6\ln|2| = 9$

$= 6\ln|x| = 9 + 6\ln|2|$

$\ln|x| = \frac{3}{2} + \ln|2|$

$\cancel{e}$ e

$x = e^{3/2 + \ln 2} = \boxed{2e^{3/2}}$

② $\int_0^x 4t^3\,dt = 27\pi$

$= t^4\Big]_0^x = 27\pi$

$= x^4 - 0^4 = 27\pi$

$= x^4 = 27\pi$

$= \boxed{x = \sqrt[4]{27\pi}}$

Find x

① $\int_2^x \frac{6}{t}\,dt = 9$

② $\int_0^x 4t^3\,dt = 27\pi$

SAME

Use your calculator on problems ~~3~~4 – 6 only.

On problems 1 and 2,

(a) Find the average value of f on the given interval. (b) Find the value of c such that $f_{AVE} = f(c)$.

1. $f(x) = (x-3)$, $[2, 5]$

$$\frac{\int_2^5 (x-3)\,dx}{5-2}$$

$$= \frac{\frac{x^2}{2} - 3x \Big]_2^5}{3}$$

$$\boxed{= \frac{\left(\frac{(5)^2}{2} - 3\cdot 5\right) - \left(\frac{(2)^2}{2} - 3(2)\right)}{3}}$$

$$\frac{\frac{25}{2} - 15 - (2-6)}{3}$$

2. $f(x) = \sqrt{x}$, $[0, 4]$

$$\frac{\int_0^4 \sqrt{x}\,dx}{4-0} = \frac{\frac{2}{3}x^{3/2}\Big]_0^4}{4}$$

$$= \frac{\frac{2}{3}(4)^{3/2} - 0}{4}$$

$$= \frac{2}{12}\cdot\sqrt{4^3} = \frac{1}{6}\sqrt{64} = \frac{8}{6}$$

$$= \boxed{\frac{4}{3}}$$

3. The table below gives values of a continuous function. Use a midpoint Riemann sum with three equal subintervals to estimate the average value of f on [20, 50].

x	20	25	30	35	40	45	50
$f(x)$	42	38	31	29	35	48	60

$$\Sigma = \frac{10\cdot f(25) + 10\cdot f(35) + 10\cdot f(45)}{50-20} = \frac{10(38+29+48)}{30}$$

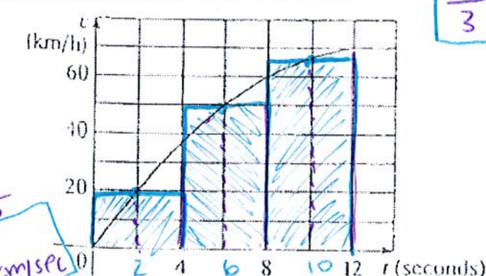
$$= \boxed{\frac{115}{3} = 38.33}$$

4. The velocity graph of an accelerating car is shown on the right.

(a) Estimate the average velocity of the car during the first 12 seconds by using a midpoint Riemann sum with three equal subintervals.

$$= \frac{4\cdot f(2) + 4\cdot f(6) + 4\cdot f(10)}{12-0}$$

$$= \frac{4\cdot 20 + 4\cdot 50 + 4\cdot 65}{12} = \frac{540}{12} = \boxed{45 \text{ km/sec}}$$



(b) At what time was the instantaneous velocity equal to the average velocity?

t = 5 seconds

5. In a certain city, the temperature, in °F, t hours after 9 AM was modeled by the function $T(t) = 50 + 14\sin\left(\frac{\pi t}{12}\right)$. Find the average temperature during the period from 9 AM to 9 PM.

$$\frac{\int_9^{21} 50 + 14\sin\left(\frac{\pi t}{12}\right) dt}{21-9} = \frac{524.373}{12} = \boxed{43.698 \text{ degrees}}$$

6. If a cup of coffee has temperature 95°C in a room where the temperature is 20°C, then, according to Newton's Law of Cooling, the temperature of the coffee after t minutes is given by the function $T(t) = 20 + 75e^{-t/50}$. What is the average temperature of the coffee during the first half hour?

$$\frac{\int_0^{30} 20 + 75e^{-t/50}\,dt}{30-0} = \frac{2291.956}{30} = \boxed{76.399°}$$

7. Suppose the $C(t)$ represents the daily cost of heating your house, measured in dollars per day, where t is time measured in days and $t = 0$ corresponds to January 1, 2010.

Interpret $\underbrace{\int_0^{90} C(t)\,dt}_{\text{The total costs to heat your house for 90 days}}$ and $\underbrace{\frac{1}{90-0}\int_0^{90} C(t)\,dt}_{}$.

The total ~~costs~~ costs to heat your house for 90 days

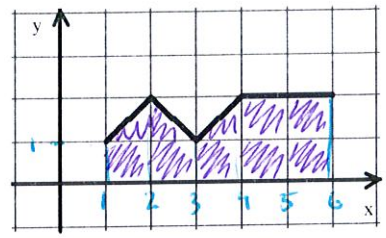
The average cost per day to heat your house for 90 days

8. Using the figure on the right,

(a) Find $\int_1^6 f(x)\,dx$. 8.5 units

(b) What is the average value of f on [1, 6]?

$\frac{8.5}{6-1} = \frac{8.5}{5} = \boxed{1.7}$



Graph of f

9. The average value of $y = f(x)$ equals 4 for $1 \le x \le 6$ and equals 5 for $6 \le x \le 8$.

What is the average value of $f(x)$ for $1 \le x \le 8$?

$20 = \frac{\boxed{\int_1^6 f(x)\,dx}}{5} = 4$ $\quad 10 = \frac{\boxed{\int_6^8 f(x)\,dx}}{2} = 5$

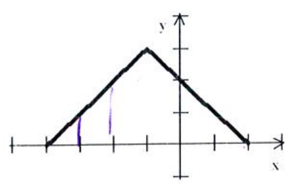
5×4
$+ 2 \times 5$
$30 \div 7 = \boxed{4.286}$

10. Suppose $\int_0^3 f(x)\,dx = 6$. What is the average value of $f(x)$ on the interval $x = 0$ to $x = 3$?

$\frac{6}{3} = \boxed{2}$

In problems 11 – 12, find the average value of the function on the given interval without integrating.
Hint: Use Geometry. (No calculator)

11. $f(x) = \begin{cases} x+4, & -4 \le x \le -1 \\ -x+2, & -1 \le x \le 2 \end{cases}$ on $[-4, 2]$

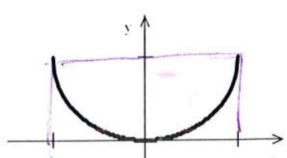


$\text{Area}_\Delta = \frac{1}{2} b h$

$= \frac{1}{2} \cdot 6 \cdot 3 = 9$

$\frac{9}{6} = \frac{3}{2}$

12. $f(x) = 1 - \sqrt{1 - x^2}$ $[-1, 1]$



$\text{Area}_\square = l \cdot w$

$= 2 \cdot 1 = 2$

$\text{Area} = \frac{1}{2}\pi r^2$

$= \frac{1}{2}\pi \cdot 1 = \frac{1}{2}\pi$

$2 - \frac{\pi}{2} = \frac{4-\pi}{2}$

$\boxed{\int_{-1}^{1} f(x)\,dx}$

$2 - \frac{1}{2}\pi$

$\frac{\frac{4-\pi}{2}}{1} = \cancel{\boxed{\frac{4-\pi}{2}}}$ $\quad 1 - \frac{\pi}{4}$

$\frac{\frac{1}{2}\pi}{2}$