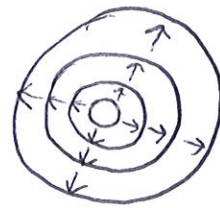


$$1. [A = \pi r^2]$$

$$\frac{dA}{dt} = \pi 2r \frac{dr}{dt}$$

↑
↑
↑
Find
50
0.01



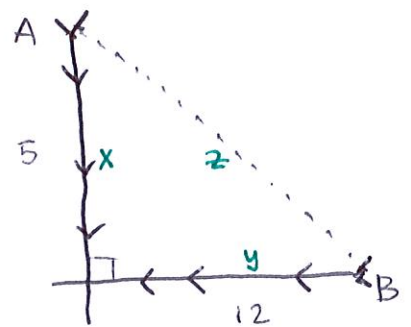
$$\frac{dA}{dt} = \pi (2) (50) (0.01) = \pi \text{ cm}^2/\text{min}$$

$$2. [x^2 + y^2 = z^2]$$

$$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 2z \frac{dz}{dt}$$

↑
↑
↑
↑
5
520
12
520
13
Find

$$\frac{dz}{dt} = \frac{5(520) + 12(520)}{13} = 680 \text{ mph}$$



$$5^2 + 12^2 = z^2$$

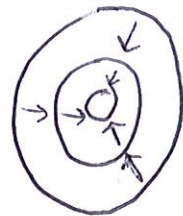
$$z^2 = 169$$

$$z = 13$$

$$3. [V = \frac{4}{3} \pi r^3]$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

↑
↑
↑
-0.08
10
Find

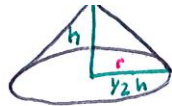


$$\frac{dr}{dt} = \frac{-0.08}{4\pi (10)^2} = -0.00006 \text{ m/min}$$

4.
$$V = \frac{1}{3} \pi r^2 h$$

$$V = \frac{1}{3} \pi \left(\frac{1}{2}h\right)^2 h$$

$$V = \frac{\pi}{12} h^3$$



height = diameter
height = 2 radius
radius = $\frac{1}{2}$ height

$$\frac{dV}{dt} = \frac{\pi}{12} \cdot 3h^2 \frac{dh}{dt}$$

↑ 2 ↑ 8 ↑ Find

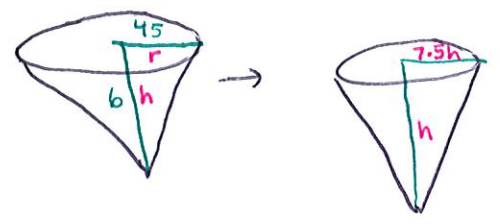
$$2 = \frac{\pi}{12} \cdot 3(8)^2 \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{2}{\frac{\pi}{12} \cdot 3 \cdot 64} = 0.0398 \text{ m/sec}$$

5.
$$V = \frac{1}{3} \pi r^2 h$$

$$V = \frac{1}{3} \pi (7.5h)^2 h$$

$$V = 18.75 \pi h^3$$



$$\frac{dV}{dt} = 56.25 \pi h^2 \frac{dh}{dt}$$

↑ -50 ↑ 5 ↑ Find

$$\frac{45}{6} = \frac{r}{h}$$

$$45h = 6r$$

$$r = \frac{45h}{6} = 7.5h$$

$$-50 = 56.25 \pi (5)^2 \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{-50}{56.25 \pi \cdot 25} = -0.011 \text{ m/min}$$

B.
$$V = \frac{1}{3} \pi r^2 h$$

$$\frac{dV}{dt} = \frac{1}{3} \pi \left(r^2 \frac{dh}{dt} + h \cdot 2r \frac{dr}{dt} \right)$$

↑ -50 ↑ 37.5 ↑ -0.011 ↑ 5 ↑ 37.5 ↑ Find

~~$$r = \frac{45h}{6} = 7.5h$$~~

$$r = 7.5h$$

$$r = 7.5(5) = 37.5$$

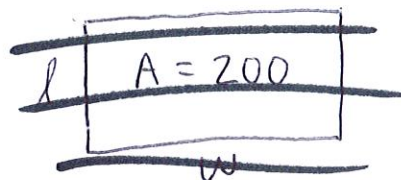
$$-50 = \frac{1}{3} \pi \left[(37.5)^2 (-0.011) + (5)(2)(37.5) \frac{dr}{dt} \right]$$

$$\frac{dr}{dt} = -0.0849 \text{ m/min}$$

~~$$6. [A = lw]$$~~

~~$$\frac{dA}{dt} = l \frac{dw}{dt} + w \frac{dl}{dt}$$~~

~~↑ ↑ ↑~~
~~200 -0.5 Find~~



~~$$A = lw$$~~
~~$$200$$~~

$$6. V = \frac{1}{3} \pi r^2 h$$

$$V = \frac{1}{3} \pi \left(\frac{1}{3}h\right)^2 h$$

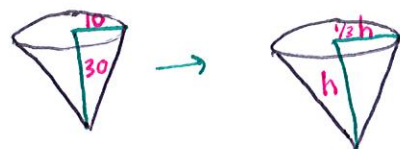
$$[V = \frac{1}{27} \pi h^3]$$

$$\frac{dV}{dt} = \frac{1}{9} \pi h^2 \frac{dh}{dt}$$

↑ ↑ ↑
 -12 20 Find

$$-12 = \frac{1}{9} \pi (20)^2 \frac{dh}{dt}$$

$$\frac{dh}{dt} = -0.0859 \text{ cm/sec}$$



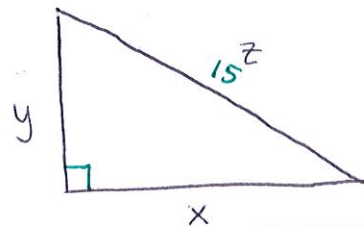
$$7. [x^2 + y^2 = z^2]$$

$$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 2z \frac{dz}{dt}$$

↑ ↑ ↑ ↑
 9 0.5 12 Find 15 0

$$(9)(0.5) + (12) \frac{dy}{dt} = (15)(0)$$

$$\frac{dy}{dt} = \frac{-(9)(0.5)}{12} = -0.375 \text{ ft/sec}$$



$$x^2 + y^2 = z^2$$

$$9^2 + y^2 = 15^2$$

$$y^2 = 144$$

$$y = 12$$

$$B. [A = \frac{1}{2} x y]$$

$$\frac{dA}{dt} = \frac{1}{2} x \frac{dy}{dt} + \frac{1}{2} y \frac{dx}{dt}$$

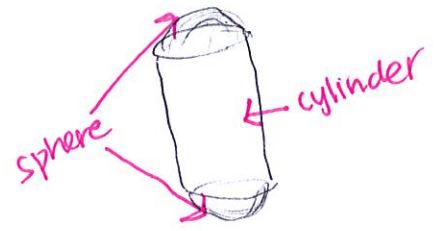
↑ ↑ ↑ ↑
 Find 9 -0.375 12 0.5

$$\frac{dA}{dt} = \frac{1}{2} (9)(-0.375) + \frac{1}{2} (12)(0.5) = 1.3125 \text{ ft}^2/\text{sec}$$

8. $V = \pi r^2 h + \frac{4}{3} \pi r^3$

A. $144\pi = \pi (3)^2 h + \frac{4}{3} \pi (3)^3$

$h = 12$ cm



B. $V = \pi r^2 h + \frac{4}{3} \pi r^3$

$\frac{dv}{dt} = \pi \left[r^2 \frac{dh}{dt} + h \cdot 2r \frac{dr}{dt} \right] + 4\pi r^2 \frac{dr}{dt}$

$261\pi = \pi \left[(3)^2 \frac{dh}{dt} + (12)(2)(3) \frac{dr}{dt} \right] + 4\pi (3)^2 (2)$

$261\pi = \pi \left[(3)^2 \frac{dh}{dt} + (12)(2)(3) \frac{dr}{dt} \right] + 4\pi (3)^2 (2)$

$\frac{dh}{dt} = 5$ cm/min

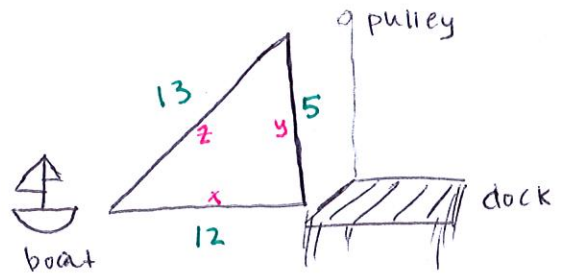
a. $x^2 + y^2 = z^2$

$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 2z \frac{dz}{dt}$

$12 \frac{dx}{dt} + 5(0) = 13(-2)$

$12 \frac{dx}{dt} + 5(0) = 13(-2)$

$\frac{dx}{dt} = \frac{13(-2)}{12} = -2.167$ m/sec



$x^2 + y^2 = z^2$

$12^2 + 5^2 = z^2$

$144 + 25 = z^2$

$z = 13$

$$10. \quad \left[V = \frac{4}{3} \pi r^3 \right]$$

A.

$$\frac{dv}{dt} = 4\pi r^2 \frac{dr}{dt}$$

↑ ↑ ↑
Find 10 0.04

$$\frac{dv}{dt} = 4\pi (10)^2 \cdot 0.04 = 50.265 \text{ cm/sec}$$

B.

$$V = \frac{4}{3} \pi r^3$$

$$36\pi = \frac{4}{3} \pi r^3$$

$$r = \sqrt[3]{27} = 3$$

$$\left[A = \pi r^2 \right]$$

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

↑ ↑ ↑
Find 3 0.04

$$\frac{dA}{dt} = 2\pi (3) (0.04) = 0.753 \text{ cm}^2/\text{sec}$$

$$V = \frac{4}{3} \pi r^3$$

$$\frac{dv}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$1 = 4\pi r^2$$

$$r^2 = \frac{1}{4\pi}$$

$$r = \sqrt{\frac{1}{4\pi}} = 0.282 \text{ cm/sec}$$

1. Eq: $z^2 = x^2 + y^2$

Der: $2z \frac{dz}{dt} = 2x \frac{dx}{dt} + 2y \frac{dy}{dt}$
13 ↑ 5 ↑ 12 ↑
 ↑ ↑ ↑ ↑
 find 2 3

$$\begin{aligned}x^2 + y^2 &= z^2 \\5^2 + 12^2 &= z^2 \\25 + 144 &= z^2 \\z^2 &= 169 \\z &= 13\end{aligned}$$

Sub: $2(13) \frac{dz}{dt} = 2(5)(2) + 2(12)(3)$

$$\frac{dz}{dt} = \frac{20 + 72}{26} = \frac{92}{26} = 3.538$$

2. Eq: $y = \sqrt{1+x^3}$

Der: $\frac{dy}{dt} = \frac{1}{2}(1+x^3)^{-1/2} \cdot 3x^2 \frac{dx}{dt}$
 ↑ ↑ ↑ ↑
 4 2 2 Find

Sub: $4 = \frac{1}{2}(1+2^3)^{-1/2} \cdot 3(2)^2 \frac{dx}{dt}$

$$\frac{dx}{dt} = \frac{4}{6(1+2^3)^{-1/2}} = \frac{4 \cdot 3}{6} = 2 \text{ cm/sec}$$

3. Eq: $PV = K$

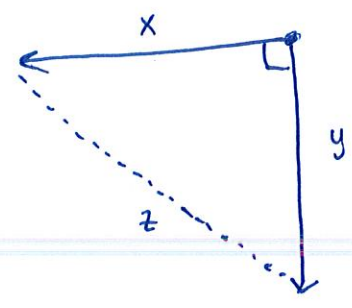
Der: $P \frac{dV}{dt} + V \frac{dP}{dt} = \frac{dK}{dt}$
 ↑ ↑ ↑ ↑
 4 5 2 0
 ↑ ↑
 find 2

Sub: $4 \frac{dV}{dt} + 5(2) = 0$

$$\frac{dV}{dt} = \frac{-10}{4} = -\frac{5}{2}$$

1. Eq: $x^2 + y^2 = z^2$

Der: $2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 2z \frac{dz}{dt}$
 ↓ 50 ↓ 25 ↓ 120 ↓ 60 ↓ 130 ↓ Find



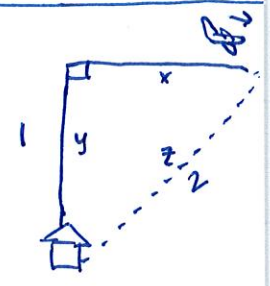
Sub: $2(50)(25) + 2(120)(60) = 130 \frac{dz}{dt}$
 $2500 + 14400 = 2(130) \frac{dz}{dt}$

$\frac{dz}{dt} = \frac{16900}{260} = 65 \text{ mph}$

$x^2 + y^2 = z^2$
 $50^2 + 120^2 = z^2$
 $2500 + 14400 = z^2$
 $z^2 = 16900$
 $z = 130$

5. Eq: $x^2 + y^2 = z^2$

Der: $2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 2z \frac{dz}{dt}$
 ↑ $\sqrt{3}$ ↑ 500 ↑ 1 ↑ 0 ↑ 2 ↑ Find



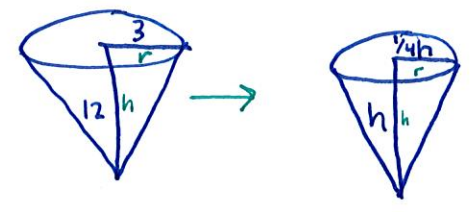
$x^2 + y^2 = z^2$
 $x^2 + 1^2 = 2^2$
 $x^2 = 4 - 1$
 $x = \sqrt{3}$

Sub: $2(\sqrt{3})(500) + 2(1)(0) = 2(2) \frac{dz}{dt}$
 $\sqrt{3} \cdot 500 = 2 \frac{dz}{dt}$

$\frac{dz}{dt} = \frac{\sqrt{3} \cdot 500}{2} = 433.013 \text{ mph}$

6. Eq: $V = \frac{1}{3} \pi r^2 h$

$V = \frac{1}{3} \pi (\frac{1}{4}h)^2 h = \frac{\pi}{48} h^3$



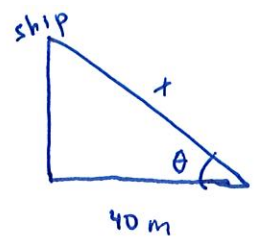
Der: $\frac{dV}{dt} = \frac{\pi}{16} h^2 \frac{dh}{dt}$
 ↑ 4 ↑ 6 ↑ Find

Sub: $4 = \frac{\pi}{16} (36) \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{4}{36\pi/16} = 0.566 \text{ ft/min}$

7. Eq: $\cos \theta = \frac{40}{x}$

Der: $-\sin \theta \frac{d\theta}{dt} = -\frac{40}{x^2} \frac{dx}{dt}$
 0.4 0.7 43.428 Find



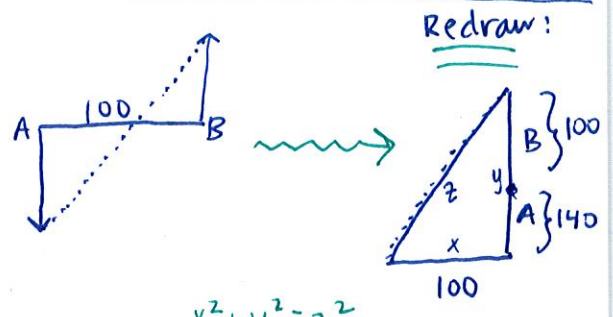
$\cos 0.4 = \frac{40}{x}$
 $x = \frac{40}{\cos 0.4} = 43.428$

Sub: $-\sin(0.4) \cdot 0.7 = \frac{-40}{(43.428)^2} \cdot \frac{dx}{dt}$

$\frac{dx}{dt} = \frac{-\sin 0.4 \cdot 0.7}{\frac{-40}{(43.428)^2}} = 12.853 \text{ rad/hr}$

8. Eq: $x^2 + y^2 = z^2$

Der: $2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 2z \frac{dz}{dt}$
 100 0 240 60 260 Find



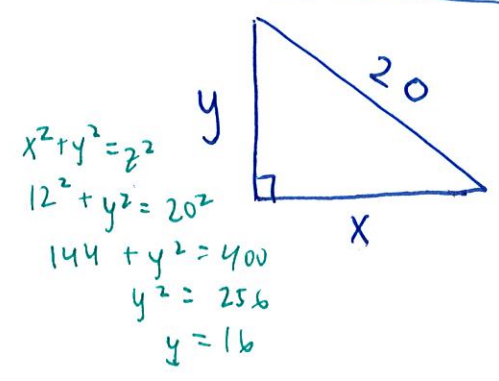
$x^2 + y^2 = z^2$
 $100^2 + 240^2 = z^2$
 $z^2 = \sqrt{67600}$
 $z = 260$

Sub: $100(0) + 240(60) = 260 \frac{dz}{dt}$

$\frac{dz}{dt} = \frac{240(60)}{260} = 55.384 \text{ km/hr}$

9. Eq: $x^2 + y^2 = z^2$

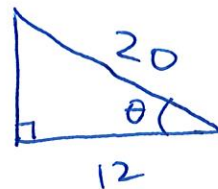
Der: $2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 2z \frac{dz}{dt}$
 12 2.5 16 Find 20 0



Sub: $12(2.5) + 16 \frac{dy}{dt} = 20(0)$

$\frac{dy}{dt} = \frac{-12(2.5)}{16} = -1.875 \text{ ft/sec}$

9. Eq: $\cos \theta = \frac{x}{20}$



Der: $-\sin \theta \frac{d\theta}{dt} = \frac{1}{20} \frac{dx}{dt}$
 0.927 ↑ Find ↑ 2.5

$\cos \theta = \frac{12}{20}$

$\theta = \arccos(\frac{12}{20})$
 $\theta = 0.927$

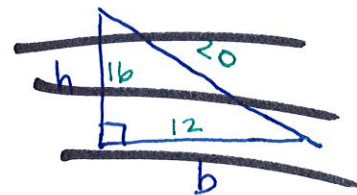
Sub: $-\sin(0.927) \cdot \frac{d\theta}{dt} = \frac{1}{20}(2.5)$

$\frac{d\theta}{dt} = \frac{(\frac{1}{20})(2.5)}{-\sin(0.927)} = -0.156 \text{ rad/sec}$

~~Eq: $A = \frac{1}{2}bh$~~

~~$A = \frac{1}{2}b \cdot \frac{4}{3}b$~~

~~$A = \frac{2}{3}b^2$~~



~~$h^2 + b^2 = 20^2$
 $h^2 + 12^2 = 400$
 $h^2 = 256$
 $h = 16$~~

~~Der: $\frac{dA}{dt} = \frac{2}{3} \cdot 2b \frac{db}{dt}$~~
 ↑ Find 12 ↑ 2.5

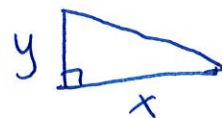
~~To avoid product rule, get h in terms of b~~

~~Sub: $\frac{dA}{dt} = \frac{2}{3} \cdot 2 \cdot 12 \cdot 2.5 = 40$~~



Eq: $A = \frac{1}{2}xy$

Der: $\frac{dA}{dt} = \frac{1}{2}x \frac{dy}{dt} + \frac{1}{2}y \frac{dx}{dt}$
 12 ↑ -1.875 ↑ 16 ↑ 2.5



Sub: $\frac{dA}{dt} = \frac{1}{2}(12)(-1.875) + \frac{1}{2}(16)(2.5) = 8.75 \text{ ft}^2/\text{sec}$