U5H2

OPTIMIZATION

1. Find two numbers whose sum is 23 and whose product is a maximum.

goal ↙ OPTIMIZE ↙ MAX

Sum: $x + y = 23$

$y = 23 - x$

PRODUCT = xy

$P = x(23-x)$

$= 23x - x^2$

$P' = 23 - 2x = 0$

$-2x = -23$

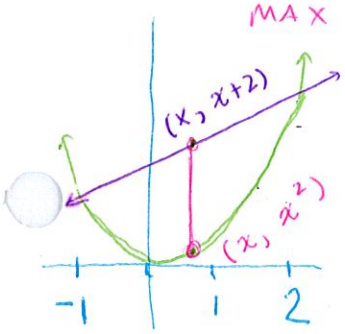
$x = 11.5$

P' : + (11.5) −

Answer: 11.5 and 11.5

2. What is maximum vertical distance between the line $y = x + 2$ and the parabola $y = x^2$ for $-1 \le x \le 2$?

MAX ↑ OPTIMIZE ↑



$d = (x+2) - (x^2)$

$= x + 2 - x^2$

$d' = 1 - 2x = 0$ IDONE

$x = 1/2$

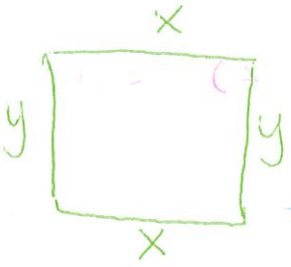
+ (1/2) −

plug $x = 1/2$ into "d" to find distance

Answer: 2.25

3. Find the dimensions of a rectangle with perimeter 100m whose area is as large as possible.

goal ↙ OPTIMIZE ↙ MAX



$P: 2x + 2y = 100$

$y = \frac{100 - 2x}{2}$

$y = 50 - x$

Area = xy

$A = x(50 - x)$

$= 50x - x^2$

$A' = 50 - 2x = 0$

$50 = 2x$

$x = 25$

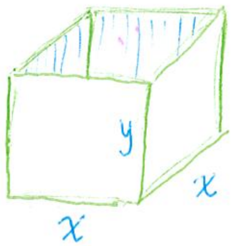
A' : + (25) −

Answer: 25 m by 25 m

4. 1200 cm^2 of material is available to make a box with a square base and an open top. Find largest possible volume of the box.

MAX

OPTIMIZE



$$SA = x^2 + 4xy = 1200$$

$$y = \frac{1200 - x^2}{4x}$$

$$y = \frac{300}{x} - \frac{x}{4}$$

$$V = x^2 y$$

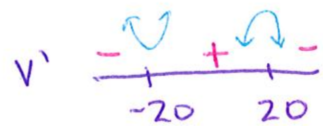
$$= x^2\left(\frac{300}{x} - \frac{x}{4}\right)$$

$$= 300x - \frac{x^3}{4}$$

$$V' = 300 - \frac{3}{4}x^2 = 0$$

$$400 = x^2$$

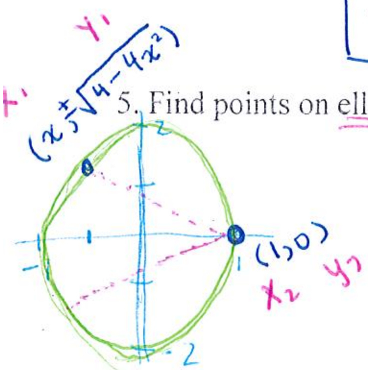
$$x = \pm 20$$



Answer: $V = 4000\ cm^3$

5. Find points on ellipse $4x^2 + y^2 = 4$ that are farthest away from point (1, 0).

OPTIMIZE + MAX



$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(x-1)^2 + (\pm\sqrt{4-4x^2})^2}$$

$$= \sqrt{x^2 - 2x + 1 + 4 - 4x^2}$$

$$= \sqrt{-3x^2 - 2x + 5}$$

$$d' = \frac{1}{2}(-3x^2 - 2x + 5)^{-1/2} \cdot (-6x - 2) = 0$$

$$\frac{-6x - 2}{2\sqrt{-3x^2 - 2x + 5}} = 0/DNE$$

$-6x - 2 = 0$

$x = -1/3$

$2\sqrt{-3x^2 - 2x + 5} = 0$

$2\sqrt{-(3x+5)(x-1)} = 0$

$x = -5/3,\ 1$

~~$2\sqrt{-3x^2 - 2x + 5} = 0$~~

~~$-3x^2 - 2x + 5 = 0$~~

~~$(3x + 5)(-x - 1)$~~

~~$x = -5/3, 1$~~

from discriminant, there are 2 x-values from denominator

Answer: $\left(-\frac{1}{3}, \pm\sqrt{\frac{32}{9}}\right)$