

U7H3 Set up integral and solve. Simplify completely so that answer matches answers on back. Show all work.

1. What is the average (mean) value of $3t^3 - t^2$ over the interval $-1 \leq t \leq 2$?

$$\frac{1}{3} \int_{-1}^2 (3t^3 - t^2) dt = \frac{1}{3} \left(\frac{3t^4}{4} - \frac{t^3}{3} \right) \Big|_{-1}^2 = \frac{1}{3} \left[\left(\frac{3(2)^4}{4} - \frac{(2)^3}{3} \right) - \left(\frac{3(-1)^4}{4} - \frac{(-1)^3}{3} \right) \right]$$

$$= \frac{1}{3} \left[\left(12 - \frac{8}{3} \right) - \left(\frac{3}{4} + \frac{1}{3} \right) \right]$$

$$= \frac{1}{3} (8.25) = \boxed{2.75 = \frac{11}{4}}$$

2. What is the average value of $f(x)$ on $[0, 4]$ if the area between the graph of $f(x)$ and the x-axis is equal to 12?

$$\frac{\int_0^4 f(x) dx}{4-0} = \bar{x} \rightarrow \frac{12}{4} = \bar{x} \rightarrow \boxed{\bar{x} = 3}$$

3. The average value of $f(x)$ on $[1, 4]$ is 5. Find $\int_1^4 f(x) dx$.

$$\frac{\int_1^4 f(x) dx}{4-1} = 5 \rightarrow \frac{\int_1^4 f(x) dx}{3} = 5 \rightarrow \int_1^4 f(x) dx = \boxed{15}$$

4. The average value of \sqrt{x} over the interval $0 \leq x \leq 2$ is what?

- a) $\frac{1}{3}\sqrt{2}$ b) $\frac{1}{2}\sqrt{2}$ **c) $\frac{2}{3}\sqrt{2}$** d) 1 e) $\frac{4}{3}\sqrt{2}$

$$\frac{\int_0^2 \sqrt{x} dx}{2-0} = \frac{1}{2} \cdot \left(\frac{2}{3} x^{3/2} \right) \Big|_0^2 = \frac{1}{2} \cdot \left[\frac{2}{3} (2)^{3/2} - \frac{2}{3} (0)^{3/2} \right]$$

$$= \frac{1}{2} \left(\frac{2}{3} \cdot \sqrt{2^3} \right)$$

$$= \frac{1}{2} \left(\frac{2}{3} \sqrt{8} \right)$$

$$= \frac{1}{3} \cdot 2\sqrt{2} = \boxed{\frac{2\sqrt{2}}{3}}$$

Find x.

5. $\int_2^x \frac{3}{x} dx = 6$

$$3 \ln x \Big|_2^x = 6$$

$$3 \ln x - 3 \ln 2 = 6$$

$$3 \ln x = 6 + 3 \ln 2$$

$$e \ln x = 2 + \ln 2$$

$$\boxed{x = e^{2 + \ln 2}}$$

6. $\int_{\pi}^x \cos x dx = 1$

$$\sin x \Big|_{\pi}^x = 1$$

$$\sin x - \sin \pi = 1$$

$$\sin x - 0 = 1$$

$$\sin x = 1$$

$$\boxed{x = \arcsin(1)}$$

7. $\int_2^x x^3 dx = 20$

$$\frac{x^4}{4} \Big|_2^x = 20$$

$$\frac{x^4}{4} - \frac{(2)^4}{4} = 20$$

$$x^4 - 16 = 80$$

$$x^4 = 96$$

$$\boxed{x = \sqrt[4]{96}}$$

Answers

1. $\frac{11}{4}$

2. 3

3. 15

4. c

5. $e^{2+\ln 2}$

6. $\frac{\pi}{2}$

7. $\sqrt[4]{96}$