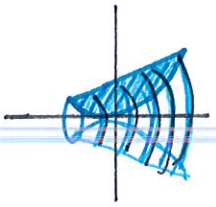
U8H3

Set up definite integral that represents each volume.

1. Rotate about x-axis

$y = x^2 + 2x + 1,\ \ y = 3x + 3$



R:

r:

Volume =

2. Rotate about y = - π

$y = x^2 + 2x + 1,\ \ y = 3x + 3$

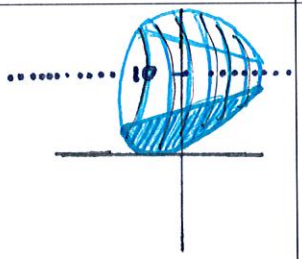
R:

r:

Volume =

3. Rotate about y = 10

$y = x^2 + 2x + 1,\ \ y = 3x + 3$



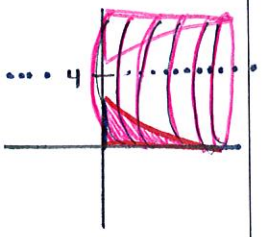
R:

r:

Volume =

4. Rotate about y = 4

$y = \dfrac{1}{1+x},\ \ y = 0,\ \ x = 3,\ \ x = 0$



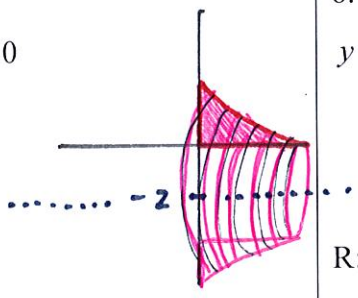
R:

r:

Volume =

5. Rotate about y = - 2

$y = \dfrac{1}{1+x},\ \ y = 0,\ \ x = 3,\ \ x = 0$



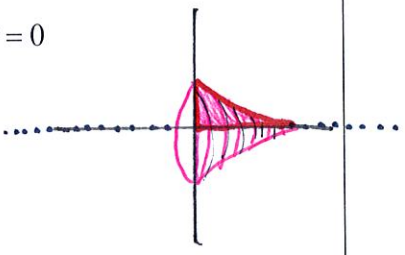
R:

r:

Volume =

6. Rotate about x- axis

$y = \dfrac{1}{1+x},\ \ y = 0,\ \ x = 3,\ \ x = 0$



R:

r:

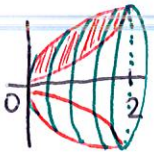
Volume =

Volumes of Solids of Revolution with holes in the middle...

Find the volume of the solid generated by revolving the region bounded by the graphs of the equations about the x-axis.

1. $y = x^2$, $y = 4x - x^2$

$x^2 = 4x - x^2$
$2x^2 - 4x = 0$
$2x(x-2) = 0$
$x = 0, 2$ → intersections (bounds)



$\pi \int_0^2 (4x - x^2)^2 - (x^2)^2 \, dx$

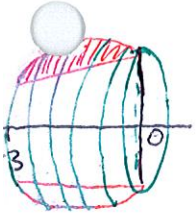
$= \pi \int_0^2 16x^2 - 8x^3 + x^4 - x^4 \, dx$

$= \pi \left[\frac{16x^3}{3} - \frac{8x^4}{4} + \frac{2x^5}{5} \right]_0^2$

$= \pi \left(\frac{16(2)^3}{3} - 2(2)^4 + \frac{2(2)^5}{5} \right) - 0 = 10\frac{2}{3}\pi$

2. $y = 6 - 2x - x^2$, $y = x + 6$

$6 - 2x - x^2 = x + 6$
$x^2 + 3x = 0$
$x(x+3) = 0$
$x = 0, -3$ → intersections (bounds)



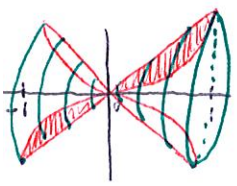
$\int_{-3}^{0} (6 - 2x - x^2)^2 - (x+6)^2 \, dx$

$= 48.6\pi$

calculator!

3. $y = x$, $y = x^3$

$x = x^3$
$x = 0, \pm 1$ → intersections (bounds)



$2\pi \int_0^1 (x)^{\frac{2}{3}} - (x^3)^2 \, dx$

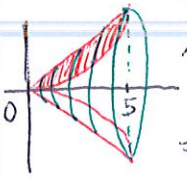
$2\pi \int_0^1 (x^2 - x^6) \, dx$

$= 2\pi \left[\frac{1}{3}x^3 - \frac{1}{7}x^7 \right]_0^1$

$= 2\pi \left[\frac{1}{3} - \frac{1}{7} - 0 \right] = .381\pi$

4. $y = 5x$, $y = x^2$

$5x = x^2$
$x^2 - 5x = 0$
$x(x-5) = 0$
$x = 0, 5$ → intersections (bounds)



$\pi \int_0^5 (5x)^2 - (x^2)^2 \, dx$

$= \pi \int_0^5 25x^2 - (x^4 - 10x^3 + 25x^2) \, dx$

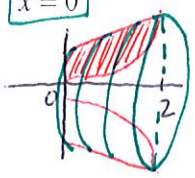
$= \pi \int_0^5 -x^4 + 10x^3 \, dx$

$= \pi \left[-\frac{x^5}{5} + \frac{10x^4}{4} \right]_0^5$

$= \pi \left(\frac{-(5)^5}{5} + \frac{10(5)^4}{4} \right) - 0 = 937.5\pi$

5. $y = x + 6$, $y = x^3$, $x = 0$

$x + 6 = x^3$
$x = 2$ → intersections (bounds)



$\pi \int_0^2 (x+6)^2 - (x^3)^2 \, dx$

$\pi \int_0^2 (x^2 + 12x + 36 - x^6) \, dx$

$= \pi \left[\frac{1}{3}x^3 + \frac{12x^2}{2} + 36x - \frac{1}{7}x^7 \right]_0^2$

$= \pi \left(\frac{1}{3}(2)^3 + 6(2)^2 + 36(2) - \frac{1}{7}(2)^7 \right)$

$= 32.381\pi$

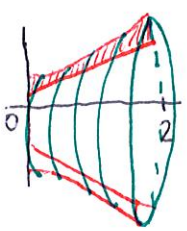
6. $y = \frac{1}{2} + x^2$, $y = x$, $x = 0$, $x = 2$ (bounds)

$\pi \int_0^2 \left(\frac{1}{2} + x^2\right)^2 - (x)^2 \, dx$

$\pi \int_0^2 \left(\frac{1}{4} + x^2 + x^4 - x^2\right) dx$

$\pi \int_0^2 \left(\frac{1}{4} + x^4\right) dx$

$= \pi \left[\frac{1}{4}x + \frac{1}{5}x^5 \right]_0^2$



$= \pi \left(\frac{1}{4}(2) + \frac{1}{5}(2)^5 \right) - 0 = 6.9\pi$

Volumes of Solids of Revolution with holes in the middle...

Find the volume of the solid generated by revolving the region bounded by the graphs of the equations about the x-axis.

1. $y = x^2$
   $y = 4x - x^2$

2. $y = 6 - 2x - x^2$
   $y = x + 6$

3. $y = x$
   $y = x^3$

4. $y = 5x$
   $y = x^2$

5. $y = x + 6$
   $y = x^3$
   $x = 0$

6. $y = \frac{1}{2} + x^2$
   $y = x$
   $x = 0$
   $x = 2$

U8H3

$3x+3 = x^2+2x+1$

$0 = x^2 - x - 2$

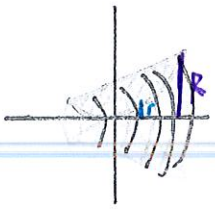
$= (x-2)(x+1)$

$x = 2, -1$

Set up definite integral that represents each volume.

1. Rotate about x-axis

$y = x^2 + 2x + 1, \quad y = 3x + 3$



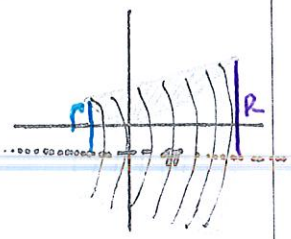
R: $3x+3$

r: x^2+2x+1

Volume = $\pi \int_{-1}^{2} (3x+3)^2 - (x^2+2x+1)^2 \, dx$

2. Rotate about y = - π

$y = x^2 + 2x + 1, \quad y = 3x + 3$



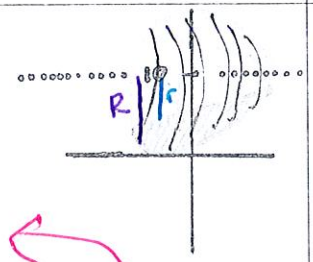
R: $3x+3+\pi$

r: $x^2+2x+1+\pi$

Volume = $\pi \int_{-1}^{2} (3x+3+\pi)^2 - (x^2+2x+1+\pi)^2 dx$

3. Rotate about y = 10

$y = x^2 + 2x + 1, \quad y = 3x + 3$



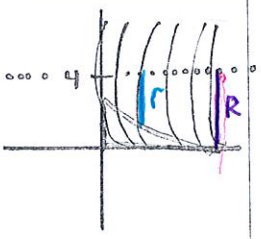
R: $10 - 3x - 3$

r: $10 - x^2 - 2x - 1$

Volume = $\pi \int_{-1}^{2} (7-3x)^2 - (9-x^2-2x)^2 \, dx$

4. Rotate about y = 4

$y = \frac{1}{1+x}, \quad y = 0, \quad x = 3, \quad x = 0$



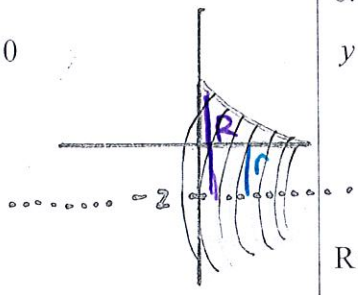
R: 4

r: $4 - \frac{1}{1+x}$

Volume = $\pi \int_{0}^{3} (4)^2 - \left(4 - \frac{1}{1+x}\right)^2 dx$

5. Rotate about y = - 2

$y = \frac{1}{1+x}, \quad y = 0, \quad x = 3, \quad x = 0$



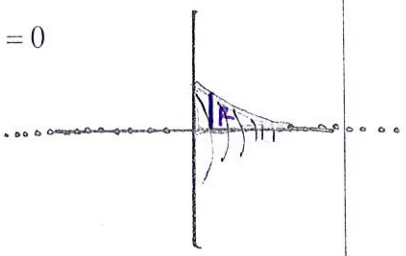
R: $\frac{1}{1+x} + 2$

r: 2

Volume = $\pi \int_{0}^{3} \left(\frac{1}{1+x} + 2\right)^2 - (2)^2 \, dx$

6. Rotate about x- axis

$y = \frac{1}{1+x}, \quad y = 0, \quad x = 3, \quad x = 0$



R: $\frac{1}{1+x} - 0$

r: 0

Volume = $\pi \int_{0}^{3} \left(\frac{1}{1+x}\right)^2 dx$