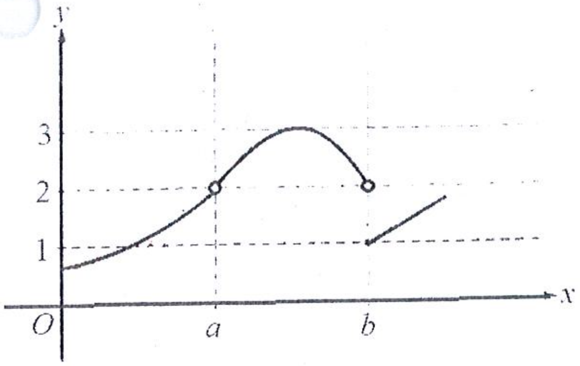
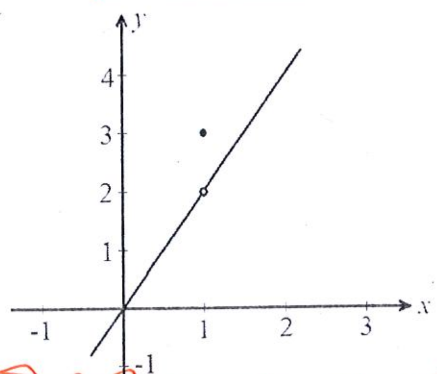
Limits and Continuity **Prep 1** Name Key Date ________

Multiple Choice

Identify the choice that best completes the statement or answers the question.



1. The graph of the function f is shown in the figure above. Which of the following statements about f is true?
   a. $\lim_{x\to a} f(x) = \lim_{x\to b} f(x)$
   b. $\lim_{x\to a} f(x) = 2$
   c. $\lim_{x\to b} f(x) = 2$
   d. $\lim_{x\to b} f(x) = 1$
   e. $\lim_{x\to a} f(x)$ does not exist



2. The graph of the function f is shown in the figure above. The value of $\lim_{x\to 1} \sin(f(x))$ is
   a. 0.909
   b. 0.841
   c. 0.141
   d. −0.416
   e. nonexistent

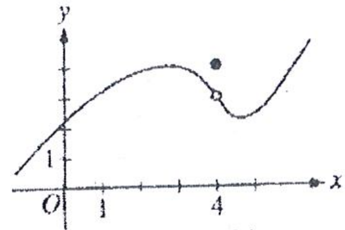
$\lim_{x\to 1} \sin(f(1)) =$

$= \sin(3) = 0.141$

(compute on calculator)

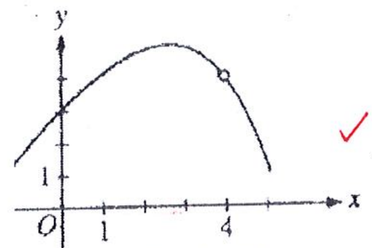
3. For which of the following does $\lim_{x\to 4} f(x)$ exist?

I.



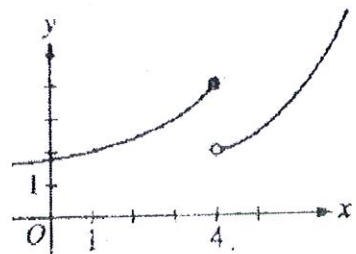
Graph of f

II.



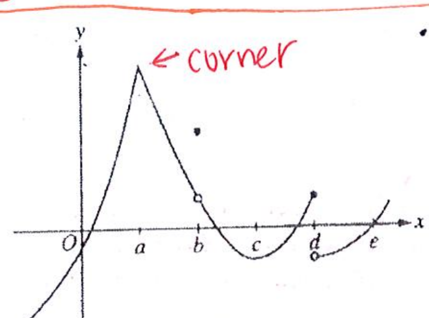
Graph of f

III.



Graph of f

a. I only b. II only c. III only
d. I and II only e. I and III only



Graph of f

4. The graph of a function f is shown above. At which value of x is f continuous, but not differentiable?
   a. a b. b c. c
   d. d e. e

5. $\lim_{x\to\infty} \dfrac{x^3 - 2x^2 + 3x - 4}{4x^3 - 3x^2 + 2x - 1} = \dfrac{1}{4}$

a. 4
b. 1
c. $\frac{1}{4}$
d. 0
e. -1

6. If $a \neq 0$, then $\lim_{x\to a} \frac{x^2 - a^2}{x^4 - a^4}$ is

use L'Hospitals OR factor

a. $\frac{1}{a^2}$
b. $\frac{1}{2a^2}$
c. $\frac{1}{6a^2}$
d. 0
e. Nonexistent

7. Find the horizontal asymptotes of the function

$\lim_{x\to\pm\infty}$ $f(x) = \frac{3}{4 - e^x}$.

a. $y = \frac{3}{4}$
b. $y = 0$
c. $y = 1$
d. $y = 3$
e. $y = 0$ and $y = \frac{3}{4}$

8. Let f be the function given by

$f(x) = \frac{(x-1)(x^2-4)}{x^2 - a}$. For what $a \geq 0$ is f continuous for all real numbers x?

a. None ✓
b. 1 only × (if a=1... hole at x=1)
c. 2 only × ~~(if a=2... f(2)=0)~~
d. 4 only × (if a=4... hole at x=±2)
e. 1 and 4 only ×

3. Let f be the function given by $f(x) = 2xe^{2x}$.

(a) Find $\lim_{x\to-\infty} f(x)$ and $\lim_{x\to\infty} f(x)$.

$= -2\infty \cdot e^{-2\infty}$

$= \frac{-2\infty}{e^{2\infty}} = \boxed{0}$

$= 2\infty \cdot e^{2\infty}$

$= \boxed{\infty}$

equation with initial condition $f(0) = -1$.

5. (a) Find $\lim_{x\to 0} \frac{f(x)+1}{\sin x}$. Show the work that leads to your answer.

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$\frac{f(0)+1}{\sin 0} = \frac{0}{0}$ (indeterminant... use L'Hospitals)

$\frac{f'(x)}{\cos x} \to \frac{f'(0)}{\cos 0} = \frac{}{1}$

9. If $f(x) = \begin{cases} \ln x & \text{for } 0 < x \leq 2 \\ x^2 \ln 2 & \text{for } 2 < x \leq 4, \end{cases}$ the $\lim_{x\to 2} f(x)$ is

Check limit from LEFT & RIGHT of $x=2$

a. $\ln 2$
b. $\ln 8$
c. $\ln 16$
d. 4
e. Nonexistent

10. $f(x) = \begin{cases} x+2 & \text{if } x \leq 3 \\ 4x - 7 & \text{if } x > 3 \end{cases}$

I. $\lim_{x\to 3} f(x)$ exists ✓
II. f is continuous at $x = 3$. ✓
III. f is differentiable at $x = 3$. × (think graphically... CORNER)

a. None
b. I only
c. II only
d. I and II only
e. I, II, and III

11. Let f be a function such that $\lim_{h\to 0} \frac{f(2+h) - f(2)}{h} = 5$.

Which of the following must be true?

I. f is continuous at $x = 2$.
II. f is differentiable at $x = 2$.
III. The derivative of f is continuous at $x = 2$.

a. I only
b. II only
c. I and II only
d. I and III only
e. II and III only

12. Find the horizontal asymptotes of the function

$\lim_{x\to\pm\infty}$ $f(x) = \frac{\sqrt{16x^2 - 5}}{2x - 6}$.

a. $y = 2$
b. $y = 8$ and $y = -8$
c. $y = 2$ and $y = -2$
d. $y = 8$
e. None

14. Let f be the function defined by

$$f(x) = \begin{cases} \sqrt{x+1} & \text{for } 0 \leq x \leq 3 \\ 5 - x & \text{for } 3 < x \leq 5. \end{cases}$$

(a) Is f continuous at $x = 3$? Explain why or why not.

$f(3) = \lim_{x\to 3} f(x)$

$2 = 2$

$\boxed{\text{yes; } f(3) = \lim_{x\to 3} f(x)}$