Calculus Worksheet review Product, Quotient, and Chain Rule

Find the first derivative of each.

1) $f(x)=(x+5)(2x-3)(3x^2+4)$ Rewrite: $f(x)=(2x^2+7x-15)(3x^2+4)$

$f'(x)=(3x^2+4)(4x+7)+(2x^2+7x-15)(6x)$

2) $f(x)=\left(\frac{2x}{x-1}\right)\left(\frac{3x+4}{5x^2-7}\right)$ Rewrite: $f(x)=\frac{2x(3x+4)}{(x-1)(5x^2-7)}=\frac{6x^2+8x}{5x^3-7x-5x^2+7}$

$f'(x)=\frac{(5x^3-7x-5x^2)(12x+8)-(6x^2+8x)(15x^2-7-10x)}{(5x^3-7x-5x^2+7)^2}$

3) $g(x)=5x^3(\sin x)^2$

$f'(x)=(\sin x)^2\cdot 15x^2+5x^3\cdot 2\sin x\cos x$

4) $t(x)=(1+\cos^2 7x)^3$ Rewrite: $f(x)=(1+[\cos(7x)]^2)^3$

$f'(x)=3(1+[\cos 7x]^2)^2\cdot 2\cos 7x\cdot -\sin 7x\cdot 7$

5) $y=x^3(2x-5)^4$

$f'(x)=(2x-5)^4\cdot 3x^2+x^3\cdot 4(2x-5)^3\cdot 2$

6) $g(x)=\left(\frac{\sin x}{1+\cos x}\right)^2$

$g'(x)=2\left(\frac{\sin x}{1+\cos x}\right)\cdot\left(\frac{(1+\cos x)(\cos x)-(\sin x)(+\sin x)}{(1+\cos x)^2}\right)$

7) $r(x)=\sqrt{x-1}+\sqrt{x+1}$ Rewrite: $y=(x-1)^{1/2}+(x+1)^{1/2}$

$y'=\frac{1}{2}(x-1)^{-1/2}+\frac{1}{2}(x+1)^{-1/2}$

8) $y=x^2\tan\frac{1}{x}$ Rewrite: $y=x^2\tan(x^{-1})$

$y'=\tan(x^{-1})\cdot 2x+x^2\cdot\sec^2(x^{-1})\cdot -1x^{-2}$

9) $k(x)=37-\sec^3(2x)$ Rewrite: $y=37-(\sec 2x)^3$

$y'=-3[\sec(2x)]^2\cdot\tan(2x)\sec(2x)\cdot 2$

10) Find the second derivative of the function $f(x)=\sin(x^2)$

$y'=\cos(x^2)\cdot 2x=2x\cdot\cos x^2$

$y''=\cos x^2\cdot 2+2x\cdot -\sin x^2\cdot 2x$

11) The position of a particle moving along a coordinate line is $s(t)=\sqrt{1+4t}$, with s in meters and t in seconds. Find the particle's velocity and acceleration at $t=6$ sec.

$v=\frac{1}{2}(1+4t)^{-1/2}\cdot 4 \rightarrow v(6)=\frac{2}{5}$ m/s

$a=2\cdot -\frac{1}{2}(1+4t)^{-3/2}\cdot 4 \rightarrow a(6)=-\frac{4}{125}$ m/s^2

True of False

12) If $y=(1-x)^{\frac{1}{2}}$, then $y'=\frac{1}{2}(1-x)^{-\frac{1}{2}}$ TRUE

13) If y is a differentiable function of u, u is a differentiable function of v, and v is a differentiable function of x, then $\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dv}\frac{dv}{dx}$ TRUE → chain rule

$\frac{dy}{dx}=\frac{dy}{dx}$

Implicit Differentiation Worksheet

Use implicit differentiation to find the derivative:

1. $x^2 - y^2 = 1$ $2x - 2y\frac{dy}{dx} = 0$ $\frac{dy}{dx} = x/y$

2. $xy = 1$ $y + x\frac{dy}{dx} = 0$ $dy/dx = -y/x$

3. $x^3 + y^3 = 1$ $3x^2 + 3y^2\frac{dy}{dx} = 0$ $dy/dx = -x^2/y^2$

4. $\sqrt{x} + \sqrt{y} = 1$ $\frac{1}{2}x^{-1/2} + \frac{1}{2}y^{-1/2}\frac{dy}{dx} = 0$ $dy/dx = \frac{x^{-1/2}}{y^{-1/2}} = \frac{y^{1/2}}{x^{1/2}}$

5. $16x^2 + 25y^2 = 400$ $32x + 50y\frac{dy}{dx} = 0$ $\frac{dy}{dx} = \frac{-32x}{50y} = -\frac{16x}{25}$

6. $x^2 + xy + y^2 = 9$ $2x + (y + x\frac{dy}{dx}) + 2y\frac{dy}{dx} = 0$ $x\frac{dy}{dx} + 2y\frac{dy}{dx} = -2x - y$ $\frac{dy}{dx}(x + 2y) = -2x - y$ $\frac{dy}{dx} = \frac{-2x - y}{x + 2y}$

Find the slope of the curve at the given point:

11. $x^2 + y^2 = 25$; $(3,-4)$

12. $xy = -8$; $(4,-2)$

Answers below

13. $x^2y = x + 2$; $(2,1)$

14. $x^{1/4} + y^{1/4} = 4$; $(16,16)$

11. $2x + 2y\frac{dy}{dx} = 0$

$\frac{dy}{dx} = -\frac{x}{y}\Big|_{(3,-4)} = \frac{3}{4}$

12. $y + x\frac{dy}{dx} = 0$

$\frac{dy}{dx} = -\frac{y}{x}\Big|_{(4,-2)} = \frac{1}{2}$

13. $y \cdot 2x + x^2\frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{1 - 2xy}{x^2}\Big|_{(2,1)} = \frac{-3}{4}$

14. $\frac{1}{4}x^{-3/4} + \frac{1}{4}y^{-3/4}\frac{dy}{dx} = 0$

$x^{-3/4} + y^{-3/4}\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{-x^{-3/4}}{y^{-3/4}} = -\frac{y^{3/4}}{x^{3/4}}$

$\frac{dy}{dx}\Big|_{(16,16)} = \frac{-16^{3/4}}{16^{3/4}} = -1$

Working with numerical values suppose that functions f and g and their derivatives have the following values at $x = 2$ and $x = 3$.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
2	8	2	1/3	-3
3	3	-4	2π	5

Evaluate the derivatives with respect to x of the following combinations at the given value of x.

14) $2f(x)$ at $x=2$

$2f'(x)$

$2f'(2)$

$2 \cdot 1/3$

$2/3$

15) $f(x) \cdot g(x)$ at $x=3$

$gf' + fg'$

$-4 \cdot 2\pi + 3 \cdot 5$

$-8\pi + 15$

16) $(f \circ g)$ at $x=2$

$f(g(x))$ at $x = 2$

$f'(g(x)) \cdot g'(x)$

$f'(g(2)) \cdot g'(2)$

$f'(2) \cdot -3$

$1/3 \cdot -3 = -1$

17) $\sqrt{f(x)}$ at $x=2$

$\frac{1}{2}(f(x))^{-1/2} \cdot f'(x)$

$\frac{1}{2}(8)^{-1/2} \cdot 1/3$

$\frac{1}{6} \cdot \frac{1}{\sqrt{8}} = \frac{1}{12\sqrt{2}}$

18) $g(f(x))$ at $x=3$

$g'(f(x)) \cdot f'(x)$

$g'(f(3)) \cdot f'(3)$

$g'(3) \cdot 2\pi$

$5 \cdot 2\pi$

10π