Cross Section Volume Worksheet: AP Calculus

Set up the integral to find the volume of the solid described by each situation. You do NOT need to solve any of these.

classwork

1. The base of the volume is a circle centered at the origin with radius 5. The cross sections perpendicular to the x-axis are:
   a. Squares
   ~~b. Equilateral triangles.~~
   c. Isosceles right triangles with leg on the base
   ~~d. Isosceles right triangles with hypotenuse on the base.~~
   e. Semi-circles
   f. Quarter-circles

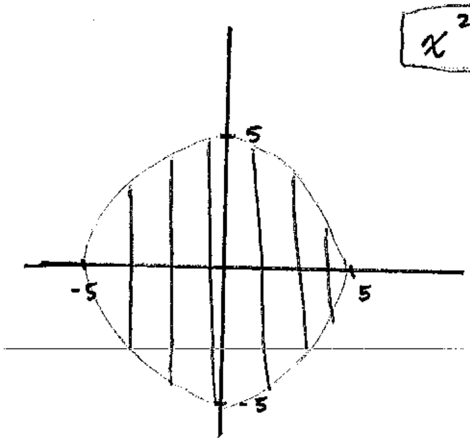
homework

2. The base of the volume is the region bounded by the curves $y = 8 - x^2$ and $y = x^2$. The cross sections perpendicular to the x-axis are:
   a. Squares
   b. Equilateral triangles
   c. Isosceles right triangles with leg on the base
   ~~d. Isosceles right triangles with hypotenuse on the base~~
   e. Semi-circles
   f. Quarter-circles

1

$$\boxed{x^2 + y^2 = 25}$$

$$\therefore \; y = \pm\sqrt{25-x^2}$$



a. $A_\square = (\text{side})^2$

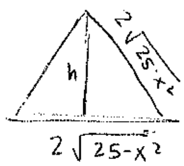
$= \left(2\sqrt{25-x^2}\right)^2$

$$\boxed{V = \int_{-5}^{5} 100 - 4x^2 \, dx}$$

b. $A_\triangle = \frac{1}{2}\,\text{base} \cdot \text{height}$

$= \frac{1}{2}\left(2\sqrt{25-x^2}\right) \cdot \left(75 - 3x^2\right)$

$$\boxed{V = \int_{-5}^{5} \left(\sqrt{25-x^2}\right)\left(75-3x^2\right) dx}$$



$h^2 + (25-x^2) = \left(2\sqrt{25-x^2}\right)^2$

$= (4(25-x^2) - (25-x^2)$

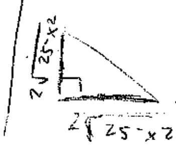
$= 100 - 4x^2 - 25 + x^2$

$= 75 - 3x^2$

c. $A_\triangle = \frac{1}{2}\,\text{base} \cdot \text{height}$

$= \frac{1}{2}\left(2\sqrt{25-x^2}\right)\left(\sqrt{800-8x^2}\right)$

$$\boxed{V = \int_{-5}^{5} \left(\sqrt{25-x^2}\right)\left(\sqrt{800-8x^2}\right) dx}$$



$4(100-x^2) + 4(100-x^2) = h^2$

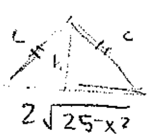
$h = \sqrt{8(100-x^2)}$

$= \sqrt{800-8x^2}$

correct first! ~~d.~~ $A_\triangle = \frac{1}{2}\,\text{base} - \text{height}$

$= \frac{1}{2}\left(2\sqrt{25-x^2}\right)\left(\sqrt{2x^2-50}\right)$

$$\boxed{V = \int_{-5}^{5} \sqrt{25-x^2}\,\sqrt{2x^2-50}\, dx}$$



$2c^2 = \left(2\sqrt{25-x^2}\right)^2$

$2c^2 = 4(25-x^2)$

$c^2 = 50 - 2x^2$

$c = \sqrt{50-2x^2}$

$50 - 2x^2 = h^2 + 4(25-x^2)$

$h^2 = 50 - 2x^2 - 100 + 4x^2$

$h = \sqrt{2x^2 - 50}$

e. $A_\circ = \frac{1}{2}\pi r^2$

$= \frac{1}{2}\pi\left(\sqrt{25-x^2}\right)^2$

$$\boxed{V = \int_{-5}^{5} \frac{1}{2}\pi\left(25-x^2\right) dx}$$

f. $A_\circ = \frac{1}{4}\pi r^2$

$= \frac{1}{4}\pi\left(2\sqrt{25-x^2}\right)^2$

$$\boxed{V = \int_{-5}^{5} \pi\left(25-x^2\right) dx}$$

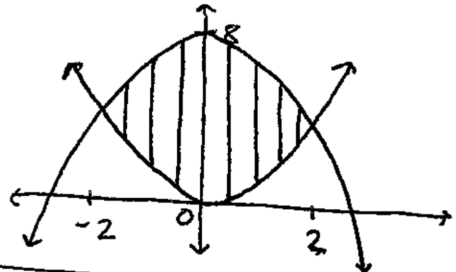
② homework

$8 - x^2 = x^2$

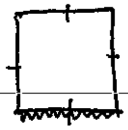
$8 = +2x^2$

$x^2 = 4$

$x = \pm 2$



a.



base
$8-2x^2$

length of base $= (8-x^2)-(x^2)$
$= 8-2x^2$

$A_{\square} = (\text{base})^2$
$= (8-2x^2)^2$

$$\boxed{V = \int_{-2}^{2} (8-2x^2)^2 \, dx}$$

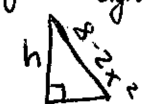
b.

base
$8-2x^2$

$A_{\triangle} = \frac{1}{2}\,\text{base}\cdot\text{height}$

$$\boxed{V = \int_{-2}^{2} \frac{1}{2}(8-2x^2)\left(\sqrt{(8-2x^2)^2 - \left(\frac{8-2x^2}{2}\right)^2}\right) dx}$$

find height

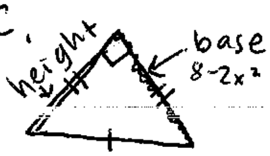


$h^2 + \left(\frac{8-2x^2}{2}\right)^2 = (8-2x^2)^2$

$h^2 = (8-2x^2)^2 - \left(\frac{8-2x^2}{2}\right)^2$

$h = \sqrt{(8-2x^2)^2 - \left(\frac{8-2x^2}{2}\right)^2}$

c.



$A_{\triangle} = \frac{1}{2}\,\text{base}\cdot\text{height}$
$= \frac{1}{2}(8-2x^2)(8-2x^2)$

$$\boxed{V = \int_{-2}^{2} \frac{1}{2}(8-2x^2)^2 \, dx}$$

d.

base
$8-2x^2$

$A_{\triangle} = \frac{1}{2}\,\text{base}\cdot\text{height}$

$$\boxed{V = \int_{-2}^{2} \frac{1}{2}(8-2x^2)\left(\sqrt{(8-2x^2)^2 - (4-x^2)^2}\right) dx}$$

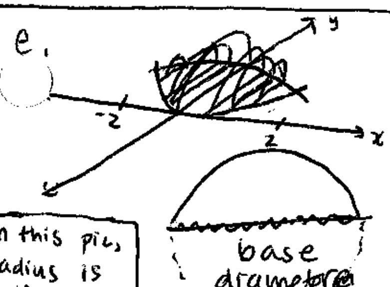
~~$2s^2 = 8-2x^2$~~ ~~$s = \sqrt{8-2x^2}$~~

~~find height~~

~~$h^2 + \sqrt{4-x^2} =$~~

$h = \sqrt{(8-2x^2)^2 - \left(\frac{8-2x^2}{2}\right)^2}$
from part b.

e.



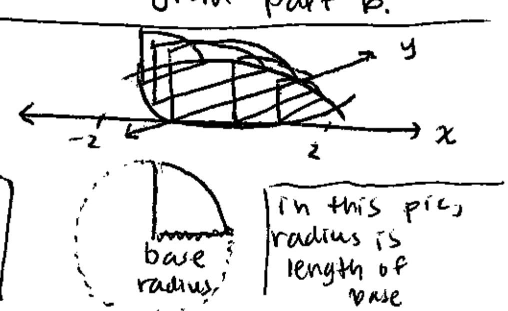
in this pic, radius is half of the base

$A_{\circ} = \frac{1}{2}\pi r^2$
$= \frac{1}{2}\pi\left(\frac{8-2x^2}{2}\right)^2$

$$\boxed{V = \int_{-2}^{2} \frac{1}{2}\pi\left(\frac{8-2x^2}{2}\right)^2 dx}$$

f. $A_{\circ} = \frac{1}{4}\pi r^2$
$= \frac{\pi}{4}(8-2x^2)^2$

$$\boxed{V = \int_{-2}^{2} \frac{1}{4}\pi(8-2x^2)^2 \, dx}$$



in this pic, radius is length of base