AP Calculus

Derivatives Practice

Name _____ Date _____

1. If $f(x) = \sin^4 x$, then $f'\left(\frac{\pi}{3}\right) =$

a) $-\frac{\sqrt{3}}{4}$ b) $9\sqrt{3}$ c) $\frac{27}{4}$ d) $\frac{\sqrt{3}}{4}$ e) $\frac{3\sqrt{3}}{4}$

2. Given $f(x) = \frac{x}{\tan x}$, find $f'\left(\frac{\pi}{2}\right)$.

a) $\frac{1}{2}$ b) $\frac{\pi}{2}$ c) $\frac{\pi}{2} - 1$ d) 4 e) undefined

3. Differentiate: $f(x) = x^2 + 2\tan x$

a) $2x + 2\tan x$ b) $2x + \sec^2 x$ c) $2 + \sec^2 x$ d) $2x + 2\sec^2 x$ e) $2x + 2\cot x$

4. Find the derivative, $\frac{dy}{dx}$, of $y = \frac{3x}{x^2 + 1}$.

a) $\frac{3}{1 + x^2}$ b) $\frac{3}{2x}$ c) $\frac{3x^2 - 3}{(1 + x^2)^3}$ d) $\frac{3(1 - x^2)}{(1 + x^2)^2}$ e) $\frac{6x + x^2}{(x^2 + 1)^2}$

5. Find the derivative, $\frac{dy}{dx}$, of $f(x) = \frac{x^2 - 1}{x^2 + 1}$.

a) $\frac{4x}{(x^2 + 1)^2}$ b) 1 c) $-\frac{4x}{(x^2 + 1)^2}$ d) $\frac{4x^2}{(x^2 + 1)^2}$ e) $\frac{-4x^2 - 4x}{(x^2 + 1)^2}$

6. If $f(x) = (x^3 + 4x^2 - 12x + 8)(3x^2 - 9x + 7)$, then find $f'(1)$.

a) -4 b) 4 c) -3 d) 3 e) 7

7. Find the slope of a line tangent to the graph of $f(x) = \frac{x+3}{x+2}$ at the point $(1, \frac{4}{3})$.

a) $-\frac{5}{9}$ b) $-\frac{1}{9}$ c) $\frac{1}{9}$ d) $\frac{5}{9}$ e) $\frac{3}{2}$

(3)

8. Find an equation of the tangent line to the curve $f(x) = x^2 - 10$ passing through the point $(5, 1)$.

a) $y - 1 = -10(x - 5)$ b) $y + 5 = -10(x + 1)$ c) $y + 1 = 10(x + 5)$
d) $y - 1 = 10(x - 5)$ e) $y - 5 = 10(x - 1)$

9. Find the slope of the tangent to the graph $f(x) = \frac{\sin x}{\cos 2x}$ where $x = \frac{\pi}{6}$.

a) $\frac{\sqrt{3}}{2}$ b) $\frac{2\sqrt{3}}{3}$ c) 3 d) $\sqrt{3}$ e) $3\sqrt{3}$

10. Find $f'(x)$ for $f(x) = (2x^2 + 5)^7$.

a) $7(4x)^6$ b) $(4x)^7$ c) $28x(2x^2 + 5)^6$ d) $7(2x^2 + 5)^6$ e) $28x^7$

11. Find $\frac{dy}{dx}$ for $y = x^3\sqrt{2x + 1}$

a) $\frac{x^2(7x + 3)}{\sqrt{2x + 1}}$ b) $\frac{3x^2}{2\sqrt{2x + 1}}$ c) $\frac{8x^3 + 3x^2}{2\sqrt{2x^4 + x^3}}$ d) $\frac{8x + 3}{\sqrt{2x + 1}}$ e) $\frac{6x^3 + 3}{\sqrt{2x + 1}}$

12. If $y = (3x^2 + 5)^5(x + 2)^4$, then $\frac{dy}{dx} =$

a) $2(x + 2)^3(3x^2 + 5)^4$ b) $2(21x^2 + 30x + 10)(x + 2)^4(3x^2 + 5)^4$
c) $(x + 2)^3(3x^2 + 5)(21x^2 + 30x + 10)$ d) $24(x + 2)^3(3x^2 + 5)^4(21x^2 + 30x + 10)$
e) $12(x + 2)^3(3x^2 + 5)^4(21x + 30)$

13. Find the derivative of $y = \cos x^3$.

a) $3x^2 \sin x^3$ b) $3\cos x^3$ c) $-3x^2 \sin x^3$ d) $3\sin x^3 \cos^2 x^3$ e) $3x\cos x^2$

14. Find $f'(x)$ given $f(x) = \sin^3(4x)$.

a) $4\cos^3(4x)$ b) $3\sin^2 4x\cos(4x)$ c) $\cos^3 4x$
d) $12\sin^2 4x\cos(4x)$ e) $12\cos^2(4x)$

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15. Find the equation of the tangent line to the graph of $3y^4 + 4x - x^2 \sin y - 4 = 0$ at the point $(1,0)$

Calculus Practice: Derivatives

Find the derivative and give the domain of the derivative for each of the following functions. If the derivative does not exist at any point, explain why and justify your answer.

1) $f(x) = x^{\frac{3}{5}} + 2$

2) $f(x) = x^{\frac{4}{7}}$

3) $f(x) = |2x+1|$

4) $f(x) = \begin{cases} x^3 \text{ if } x \le 0 \\ 2x \text{ if } x > 0 \end{cases}$

Find the equation of the tangent line to the graph of $f(x)$ at the point P.

5) $f(x) = \frac{2}{x^2}$ $P(1,2)$

6) $f(x) = x^{\frac{2}{3}} + 2x$ $P(8,20)$

8) Find the x values of all points on the graph of $y = x^4 - 2x^2$ where the tangent line is horizontal.

AP Calculus Practice (3.1-3.3)

For each of the following functions, find the derivative. For each derivative, determine all values for which the derivative does not exist. For each of these values determine if the derivative does not exist due to a discontinuity, a corner point, a cusp, or a vertical tangent line.

1) $f(x) = \frac{3x^2}{x^3+1}$

2) $f(x) = 4x^3(3x^4 - 1)$

3) $f(x) = \frac{x^1}{2x^5+3}$

Find the equation of the tangent line to the graph of the given function at the given point.

7) $f(x) = \sqrt[3]{x^2} + \frac{3}{x^2} - 2$ at $x = 1$

Find the equation of the normal line to the graph of the given function at the given point.

9) $f(x) = \sqrt[3]{x^6} - \frac{2}{x^3} - 5$ at $x = -1$

Calculus Practice: Derivatives

Find the derivative and give the domain of the derivative for each of the following functions. If the derivative does not exist at any point, explain why and justify your answer.

1) $f(x)=\sqrt[3]{x^5}+2$

$f'(x)=\frac{3}{5}x^{-2/5}$

2) $f(x)=\sqrt[4]{x^7}$

$f'(x)=\frac{4}{7}x^{-3/7}$

3) ~~$f(x)=|2x+1|$~~

4) $f(x)=\begin{cases} x^3 \text{ if } x\le 0 \\ 2x \text{ if } x>0 \end{cases}$

$f'(x)=\begin{cases} 3x^2, & x<0 \\ 2, & x>0 \end{cases}$

Find the equation of the tangent line to the graph of $f(x)$ at the point P.

5) $f(x)=\frac{2}{x^2}$ $P(1,2)$ — point: $P(1,2)$

slope: $f'(x)=-4x^{-3}$, $f'(1)=-4$

$y-2=-4(x-1)$

6) $f(x)=x^3+2x$ $P(8,20)$

slope: $f'(x)=\frac{2}{3}x^{-1/3}+2$, $f'(8)=7/3$

$y-20=\frac{7}{3}(x-8)$

8) Find the x values of all points on the graph of $y=x^4-2x^2$ where the tangent line is horizontal. ← means "where slope = 0"

$y'=4x^3-4x=0$

$4x(x^2-1)=0$

$4x(x+1)(x-1)=0$

$x=0,\pm 1$

(1)

AP Calculus Practice (3.1-3.3)

For each of the following functions, find the derivative. For each derivative, determine all values for which the derivative does not exist. For each of those values determine if the derivative does not exist due to a discontinuity, a corner point, a cusp, or a vertical tangent line.

1) $f(x)=\frac{3x^3}{x^3+1}$

$f'(x)=\frac{(x^3+1)(2x^{-1/3})+(3x^{2/3})(3x^2)}{(x^3+1)^2}$

2) $f(x)=4x^4\sqrt[3]{(3x^3-1)}$

$f'(x)=(3x^4-1)(16x^3)+(4x^4)(12x^3)$

3) $f(x)=\frac{x^5}{2x^5+3}$

$f'(x)=\frac{(2x^5+3)(5x^4)-(x^5)(10x^4)}{(2x^5+3)^2}$

Find the equation of the tangent line to the graph of the given function at the given point.

7) $f(x)=\sqrt[3]{x^2}+\frac{3}{x^2}-2$ at $x=1$ — point: $(1,2)$

slope: $f'(x)=\frac{2}{3}x^{-1/3}-6x^{-3}$, $f'(1)=-16/3$

$y-2=\frac{-16}{3}(x-1)$

Find the equation of the normal line to the graph of the given function at the given point.

↖ means "$\perp$"

9) $f(x)=\sqrt[3]{x^6}-\frac{2}{x^3}-5$ at $x=-1$

point $(-1,-2)$

slope: $f'(x)=\frac{6}{5}x^{1/5}+6x^{-4}$, $f'(-1)=24/5$ ← TANGENT SLOPE

NORMAL SLOPE $=-\frac{5}{24}$

$y+2=\frac{24}{5}(x+1)$

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a) $-\frac{\sqrt{3}}{4}$ b) $9\sqrt{3}$ c) $\frac{27}{4}$ d) $\frac{\sqrt{3}}{4}$ e) $\frac{3\sqrt{3}}{4}$

$f'(x) = 4(\sin x)^3 \cdot \cos x$

$f'(\pi/3) = 4(\sin \pi/3)^3 \cdot \cos \pi/3 = 4\left(\frac{3\sqrt{3}}{8}\right) \cdot \frac{1}{2} = \frac{3}{4}\sqrt{3}$

2. Given $f(x) = \frac{x}{\tan x}$, find $f'\left(\frac{\pi}{2}\right)$.

a) $\frac{1}{2}$ b) $\frac{\pi}{2}$ c) $\frac{\pi}{2} - 1$ d) 4 e) undefined

$f'(x) = \frac{(\tan x)(1) - (x)(\sec^2 x)}{\tan^2 x}$ $f'(\pi/2) = \frac{(0)(1) - (\pi/2)(\text{DNE})}{(\text{DNE})^2}$

3. Differentiate: $f(x) = x^2 + 2\tan x$

a) $2x + 2\tan x$ b) $2x + \sec^2 x$ c) $2 + \sec^2 x$ d) $2x + 2\sec^2 x$ e) $2x + 2\cot x$

$f'(x) = 2x + 2 \cdot \sec^2 x$

4. Find the derivative, $\frac{dy}{dx}$, of $y = \frac{3x}{x^2+1}$.

a) $\frac{3}{1+x^2}$ b) $\frac{3}{2x}$ c) $\frac{3x^2-3}{(1+x^2)^3}$ d) $\frac{3(1-x^2)}{(1+x^2)^2}$ e) $\frac{6x+x^2}{(x^2+1)^2}$

$y' = \frac{(x^2+1)(3) - (3x)(2x)}{(x^2+1)^2} = \frac{3x^2+3-6x^2}{(x^2+1)^2} = \frac{-3x^2+3}{(x^2+1)^2} = \frac{-3(x^2-1)}{(x^2+1)^2}$

5. Find the derivative, $\frac{dy}{dx}$, of $f(x) = \frac{x^2-1}{x^2+1}$.

a) $\frac{4x}{(x^2+1)^2}$ b) 1 c) $-\frac{4x}{(x^2+1)^2}$ d) $\frac{4x^2}{(x^2+1)^2}$ e) $\frac{-4x^2-4x}{(x^2+1)^2}$

$f'(x) = \frac{(x^2+1)(2x) - (x^2-1)(2x)}{(x^2+1)^2} = \frac{2x^3+2x-2x^3+2x}{(x^2+1)^2} = \frac{4x}{(x^2+1)^2}$

6. If $f(x) = (x^3 + 4x^2 - 12x + 8)(3x^2 - 9x + 7)$, then find $f'(1)$.

a) -4 b) 4 c) -3 d) 3 e) 7

$f'(x) = (3x^2+8x-12)(3x^2-9x+7) + (x^3+4x^2-12x+8)(6x-9)$

$f'(1) = (3+8-12)(3-9+7) + (1+4-12+8)(6-9) = (-1)(1) + (1)(-3) = -1-3 = -4$

7. Find the slope of a line tangent to the graph of $f(x) = \frac{x+3}{x+2}$ at the point $(1, \frac{4}{3})$.

a) $-\frac{5}{9}$ b) $-\frac{1}{9}$ c) $\frac{1}{9}$ d) $\frac{5}{9}$ e) $\frac{3}{2}$

$f'(x) = \frac{(x+2)(1) - (x+3)(1)}{(x+2)^2}$

$f'(1) = \frac{(3) - (4)}{(3)^2} = \frac{-1}{9}$

(3)

8. Find an equation of the tangent line to the curve $f(x) = x^2 - 10$ passing through the point $(5, 1)$.

a) $y - 1 = -10(x - 5)$ b) $y + 5 = -10(x + 1)$ c) $y + 1 = 10(x + 5)$

d) $y - 1 = 10(x - 5)$ e) $y - 5 = 10(x - 1)$

Slope: $f'(x) = 2x$

$f'(5) = 10$ → $y - 1 = 10(x - 5)$

9. Find the slope of the tangent to the graph $f(x) = \frac{\sin x}{\cos 2x}$ where $x = \frac{\pi}{6}$.

a) $\frac{\sqrt{3}}{2}$ b) $\frac{2\sqrt{3}}{3}$ c) 3 d) $\sqrt{3}$ e) $3\sqrt{3}$

$f'(x) = \frac{\cos(2x)(\cos x) - (\sin x)(-2\sin 2x)}{(\cos 2x)^2}$ $f'(\pi/6) = \frac{(\frac{1}{2})(\frac{\sqrt{3}}{2}) - (\frac{1}{2})(-$ $(\sqrt{3}/2)^2$

10. Find $f'(x)$ for $f(x) = (2x^2 + 5)^7$.

a) $7(4x)^6$ b) $(4x)^7$ c) $28x(2x^2 + 5)^6$ d) $7(2x^2 + 5)^6$ e) $28x^7$

$f'(x) = 7(2x^2+5)^6 \cdot (4x)$

$= 28x(2x^2+5)^6$

11. Find $\frac{dy}{dx}$ for $y = x^3\sqrt{2x+1}$

a) $\frac{x^2(7x+3)}{\sqrt{2x+1}}$ b) $\frac{3x^2}{2\sqrt{2x+1}}$ c) $\frac{8x^3+3x^2}{2\sqrt{2x^4+x^3}}$ d) $\frac{8x+3}{\sqrt{2x+1}}$ e) $\frac{6x^3+3}{\sqrt{2x+1}}$

$y' = (\sqrt{2x+1})(3x^2) + (x^3)\left(\frac{1}{2}(2x+1)^{-1/2} \cdot 2\right) = \frac{(2x+1)\cdot 3x^2}{\sqrt{2x+1}} + \frac{x^3}{\sqrt{2x+1}}$

12. If $y = (3x^2 + 5)^5(x + 2)^4$, then $\frac{dy}{dx} =$

a) $2(x + 2)^3(3x^2 + 5)^4$

b) $2(21x^2 + 30x + 10)(x + 2)^3(3x^2 + 5)^4$

c) $(x + 2)^3(3x^2 + 5)(21x^2 + 30x + 10)$

d) $24(x + 2)^3(3x^2 + 5)^4(21x^2 + 30x + 10)$

e) $12(x + 2)^3(3x^2 + 5)^4(21x + 30)$

$y' = (x+2)^4 \cdot 5(3x^2+5)^4 \cdot 6x + (3x^2+5)^5 \cdot 4(x+2)^3$

$= (x+2)^3(3x^2+5)^4\left[5(x+2) + 4(3x^2+5)\right]$

13. Find the derivative of $y = \cos x^3$.

a) $3x^2 \sin x^3$ b) $3\cos x^3$ c) $-3x^2 \sin x^3$ d) $3\sin x^3 \cos^2 x^3$ e) $3x\cos x^2$

$y = \cos(x^3)$

$y' = -\sin(x^3) \cdot 3x^2$

14. Find $f'(x)$ given $f(x) = \sin^3(4x)$.

a) $4\cos^3(4x)$ b) $3\sin^2 4x\cos(4x)$ c) $\cos^3 4x$

d) $12\sin^2 4x\cos(4x)$ e) $12\cos^2(4x)$

$f(x) = (\sin(4x))^3$

$f'(x) = 3(\sin(4x))^2 \cdot \cos(4x) \cdot 4$

(4)

15. Find the equation of the tangent line to the graph of $3y^4 + 4x - x^2\sin y - 4 = 0$ at the point $(1,0)$

$12y^3\frac{dy}{dx} + 4 - \left[(\sin y)(2x) + (x^2)(\cos y)\frac{dy}{dx}\right] = 0$

$12y^3\frac{dy}{dx} - x^2\cos y\frac{dy}{dx} = -4 + 2x\sin y$

$\frac{dy}{dx} = \frac{-4+2x\sin y}{-12y^3 - x^2}\Big|_{(1,0)} = 4$ slope

$y - 0 = 4(x - 1)$