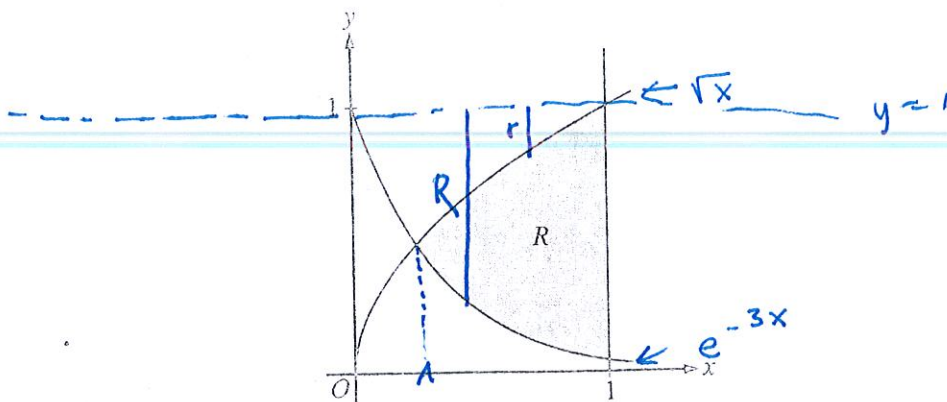


A graphing calculator is required for some problems or parts of problems.



1. Let  $R$  be the shaded region bounded by the graphs of  $y = \sqrt{x}$  and  $y = e^{-3x}$  and the vertical line  $x = 1$ , as shown in the figure above.
- Find the area of  $R$ .
  - Find the volume of the solid generated when  $R$  is revolved about the horizontal line  $y = 1$ .
  - The region  $R$  is the base of a solid. For this solid, each cross section perpendicular to the  $x$ -axis is a rectangle whose height is 5 times the length of its base in region  $R$ . Find the volume of this solid.

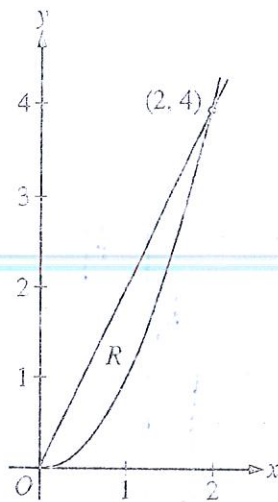
a. let  $A = 0.2387$

$$\int_A^1 \sqrt{x} - e^{-3x} dx = 0.442$$

b. 
$$\pi \int_A^1 (1 - e^{-3x})^2 - (1 - \sqrt{x})^2 dx = 0.725$$

c. 
$$\int_A^1 5(\sqrt{x} - e^{-3x})^2 dx = 1.554$$

No calculator is allowed for these problems.



4. Let  $R$  be the region in the first quadrant enclosed by the graphs of  $y = 2x$  and  $y = x^2$ , as shown in the figure above.

$$x = y/2 \quad x = \sqrt{y}$$

(a) Find the area of  $R$ .

(b) The region  $R$  is the base of a solid. For this solid, at each  $x$  the cross section perpendicular to the  $x$ -axis has area  $A(x) = \sin\left(\frac{\pi}{2}x\right)$ . Find the volume of the solid.

(c) Another solid has the same base  $R$ . For this solid, the cross sections perpendicular to the  $y$ -axis are squares. Write, but do not evaluate, an integral expression for the volume of the solid.

$$\begin{aligned} \text{a. } A &= \int_0^2 2x - x^2 \, dx && \underline{\text{OR}} \quad \int_0^4 \sqrt{y} - y/2 \, dy \\ &= \left[ x^2 - \frac{1}{3}x^3 \right]_0^2 \\ &= \left( 4 - \frac{8}{3} \right) - (0) = \frac{4}{3} \end{aligned}$$

$$\begin{aligned} \text{b. } V &= \int_0^2 \text{Area} \, dx = \int_0^2 \sin\left(\frac{\pi}{2}x\right) \, dx \\ &= \left[ -\frac{2}{\pi} \cos\left(\frac{\pi}{2}x\right) \right]_0^2 \\ &= \left( -\frac{2}{\pi} \cos \pi \right) - \left( -\frac{2}{\pi} \cos 0 \right) \\ &= \left( -\frac{2}{\pi} \cdot -1 \right) - \left( -\frac{2}{\pi} \cdot 1 \right) = \frac{2}{\pi} + \frac{2}{\pi} = \frac{4}{\pi} \end{aligned}$$

$$\text{c. } V = \int_0^4 (\sqrt{y} - y/2)^2 \, dy$$