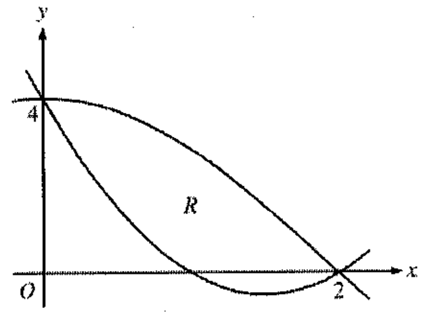
NO CALCULATOR

AP® CALCULUS AB
2013 SCORING GUIDELINES

Question 5



Let $f(x) = 2x^2 - 6x + 4$ and $g(x) = 4\cos\left(\frac{1}{4}\pi x\right)$. Let R be the region bounded by the graphs of f and g, as shown in the figure above.

(a) Find the area of R.

(b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 4$.

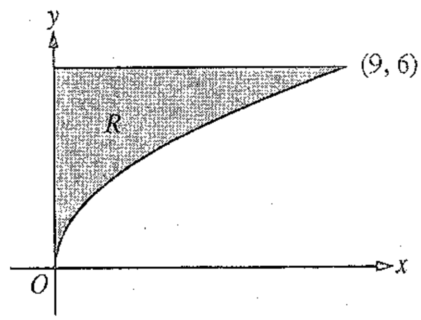
(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Write, but do not evaluate, an integral expression that gives the volume of the solid.

Solution	Scoring
(a) $\text{Area} = \int_0^2 [g(x) - f(x)]\,dx = \int_0^2 \left[4\cos\left(\frac{\pi}{4}x\right) - \left(2x^2 - 6x + 4\right)\right] dx = \left[4\cdot\frac{4}{\pi}\sin\left(\frac{\pi}{4}x\right) - \left(\frac{2x^3}{3} - 3x^2 + 4x\right)\right]_0^2 = \frac{16}{\pi} - \left(\frac{16}{3} - 12 + 8\right) = \frac{16}{\pi} - \frac{4}{3}$	4 : { 1 : integrand; 2 : antiderivative; 1 : answer }
(b) $\text{Volume} = \pi\int_0^2 \left[(4 - f(x))^2 - (4 - g(x))^2\right] dx = \pi\int_0^2 \left[\left(4 - \left(2x^2 - 6x + 4\right)\right)^2 - \left(4 - 4\cos\left(\frac{\pi}{4}x\right)\right)^2\right] dx$	3 : { 2 : integrand; 1 : limits and constant }
(c) $\text{Volume} = \int_0^2 [g(x) - f(x)]^2\,dx = \int_0^2 \left[4\cos\left(\frac{\pi}{4}x\right) - \left(2x^2 - 6x + 4\right)\right]^2 dx$	2 : { 1 : integrand; 1 : limits and constant }

NO CALCULATOR

AP® CALCULUS AB
2010 SCORING GUIDELINES

Question 4



Let R be the region in the first quadrant bounded by the graph of $y = 2\sqrt{x}$, the horizontal line $y = 6$, and the y-axis, as shown in the figure above.

(a) Find the area of R.

(b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 7$.

(c) Region R is the base of a solid. For each y, where $0 \le y \le 6$, the cross section of the solid taken perpendicular to the y-axis is a rectangle whose height is 3 times the length of its base in region R. Write, but don't evaluate, an integral that gives volume.

(a) Area $= \int_0^9 \left(6 - 2\sqrt{x}\right) dx = \left(6x - \frac{4}{3}x^{3/2}\right)\Big\vert_{x=0}^{x=9} = 18$	$3: \begin{cases} 1: \text{integrand} \\ 1: \text{antiderivative} \\ 1: \text{answer} \end{cases}$
(b) Volume $= \pi \int_0^9 \left(\left(7 - 2\sqrt{x}\right)^2 - (7-6)^2\right) dx$	$3: \begin{cases} 2: \text{integrand} \\ 1: \text{limits and constant} \end{cases}$
(c) Solving $y = 2\sqrt{x}$ for x yields $x = \frac{y^2}{4}$. Each rectangular cross section has area $\left(3\frac{y^2}{4}\right)\left(\frac{y^2}{4}\right) = \frac{3}{16}y^4$. Volume $= \int_0^6 \frac{3}{16}y^4\, dy$	$3: \begin{cases} 2: \text{integrand} \\ 1: \text{answer} \end{cases}$

CALCULATOR

AP® CALCULUS AB
2014 SCORING GUIDELINES

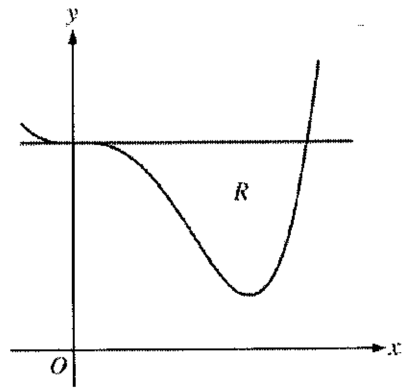
Question 2

Let R be the region enclosed by the graph of $f(x) = x^4 - 2.3x^3 + 4$ and the horizontal line $y = 4$, as shown in the figure above.

(a) Find the volume of the solid generated when R is rotated about the horizontal line $y = -2$.

(b) Region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is an isosceles right triangle with a leg in R. Find the volume of the solid.

(c) The vertical line $x = k$ divides R into two regions with equal areas. Write, but do not solve, an equation involving integral expressions whose solution gives the value k.



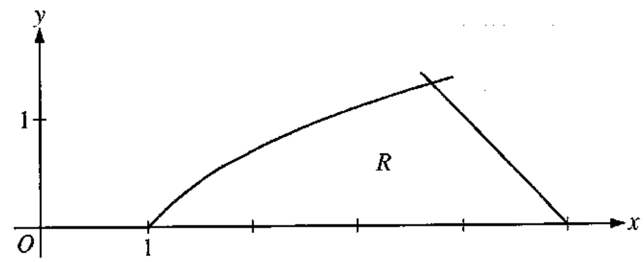
(a) $f(x) = 4 \Rightarrow x = 0, 2.3$ $\text{Volume} = \pi\int_0^{2.3}\left[(4+2)^2 - (f(x)+2)^2\right]dx$ $= 98.868$ (or 98.867)	$4 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{limits} \\ 1 : \text{answer} \end{cases}$
(b) $\text{Volume} = \int_0^{2.3} \frac{1}{2}(4 - f(x))^2\,dx$ $= 3.574$ (or 3.573)	$3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$
(c) $\int_0^k (4 - f(x))\,dx = \int_k^{2.3} (4 - f(x))\,dx$	$2 : \begin{cases} 1 : \text{area of one region} \\ 1 : \text{equation} \end{cases}$

CALCULATOR

AP® CALCULUS AB
2012 SCORING GUIDELINES

Question 2

Let R be the region in the first quadrant bounded by the x-axis and the graphs of $y = \ln x$ and $y = 5 - x$, as shown in the figure above.



(a) Find the area of R.

(b) Region R is the base of a solid. For the solid, each cross section perpendicular to the x-axis is a square. Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid.

(c) The horizontal line $y = k$ divides R into two regions of equal area. Write, but do not solve, an equation involving one or more integrals whose solution gives the value of k.

$\ln x = 5 - x \Rightarrow x = 3.69344$

Therefore, the graphs of $y = \ln x$ and $y = 5 - x$ intersect in the first quadrant at the point $(A, B) = (3.69344, 1.30656)$.

(a) Area $= \int_0^B \left(5 - y - e^y\right) dy$ $= 2.986$ (or 2.985) OR Area $= \int_1^A \ln x \, dx + \int_A^5 (5 - x) \, dx$ $= 2.986$ (or 2.985)	3 : { 1 : integrand 1 : limits 1 : answer
(b) Volume $= \int_1^A (\ln x)^2 \, dx + \int_A^5 (5 - x)^2 \, dx$	3 : { 2 : integrands 1 : expression for total volume
(c) $\int_0^k \left(5 - y - e^y\right) dy = \frac{1}{2} \cdot 2.986 \left(\text{or } \frac{1}{2} \cdot 2.985\right)$	3 : { 1 : integrand 1 : limits 1 : equation